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UNIVERSITÄT BAYREUTH

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Complexity Measures as Tools for Delineating the Possible from the Impossible in Catchment Modelling

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Introduction

- First-order catchments are appropriate for both runoff measurements and forest monitoring
- Modelling approaches in hydrology and forest science are rather different
- can they inform each other or are they mutually incompatible?
- **Goal:** integrate typical modelling perspectives in hydrology and forest science
- **Method:** characterize data sets by complexity measures

Key Monitoring Results

- runoff responds to catchment-wide events (liming, atmospheric deposition)
- SO_4 (Fig.2) budgets differ and show decreasing trends
- stand biomass indicates spatial homogeneous site quality
- tree growth identifies site quality, but it changes between subsequent rotation periods (Fig.3; monitored 1889-1914)

Study Area

- four (sub)catchments in the Harz Mountains: Lange Bramke (spring area, weir), Dicke Bramke and Steile Bramke
- Hydrology monitored since 1948, atmospheric deposition, water chemistry since the 1980ies, forestry monitored 1988-2023
- forest stand killed by bark beetle since 2021

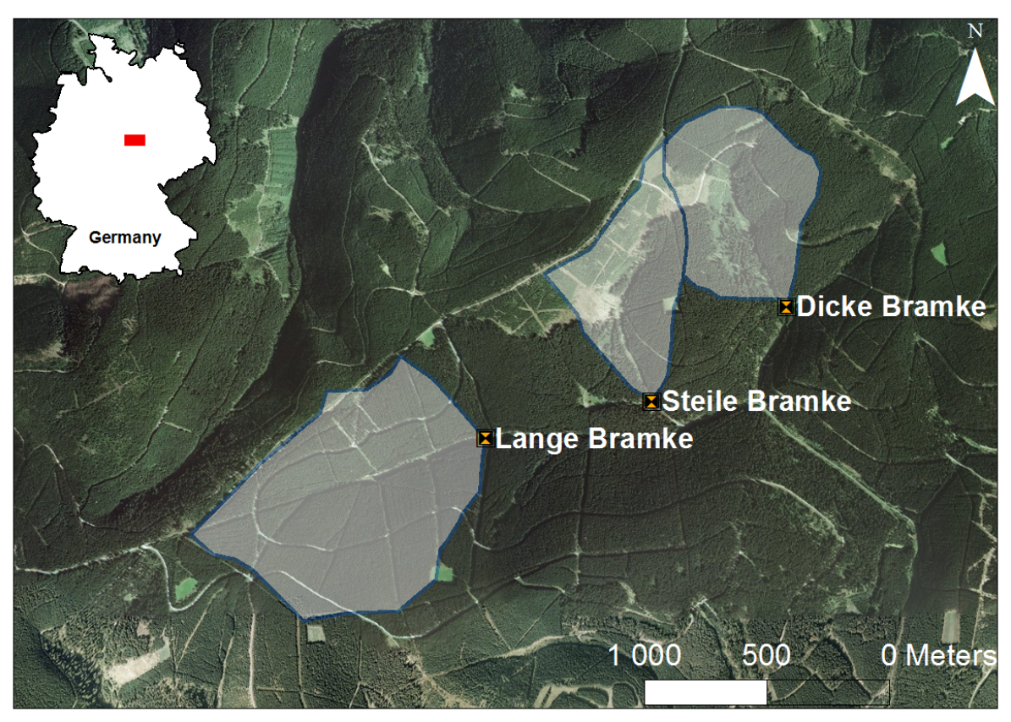


Figure 1: Study area

Data

- four catchments, weekly to biweekly sampling
- Variables: pH, Ca^{2+}, Mg^{2+}, Na^{+}, K^{+}, Cl^{-}, NO_3^{-}, SO_4^{2-}
- repeated inventories of standing timber volume
- analysed period: 1980ies–2020

Empirical Interpretation of Catchment Response

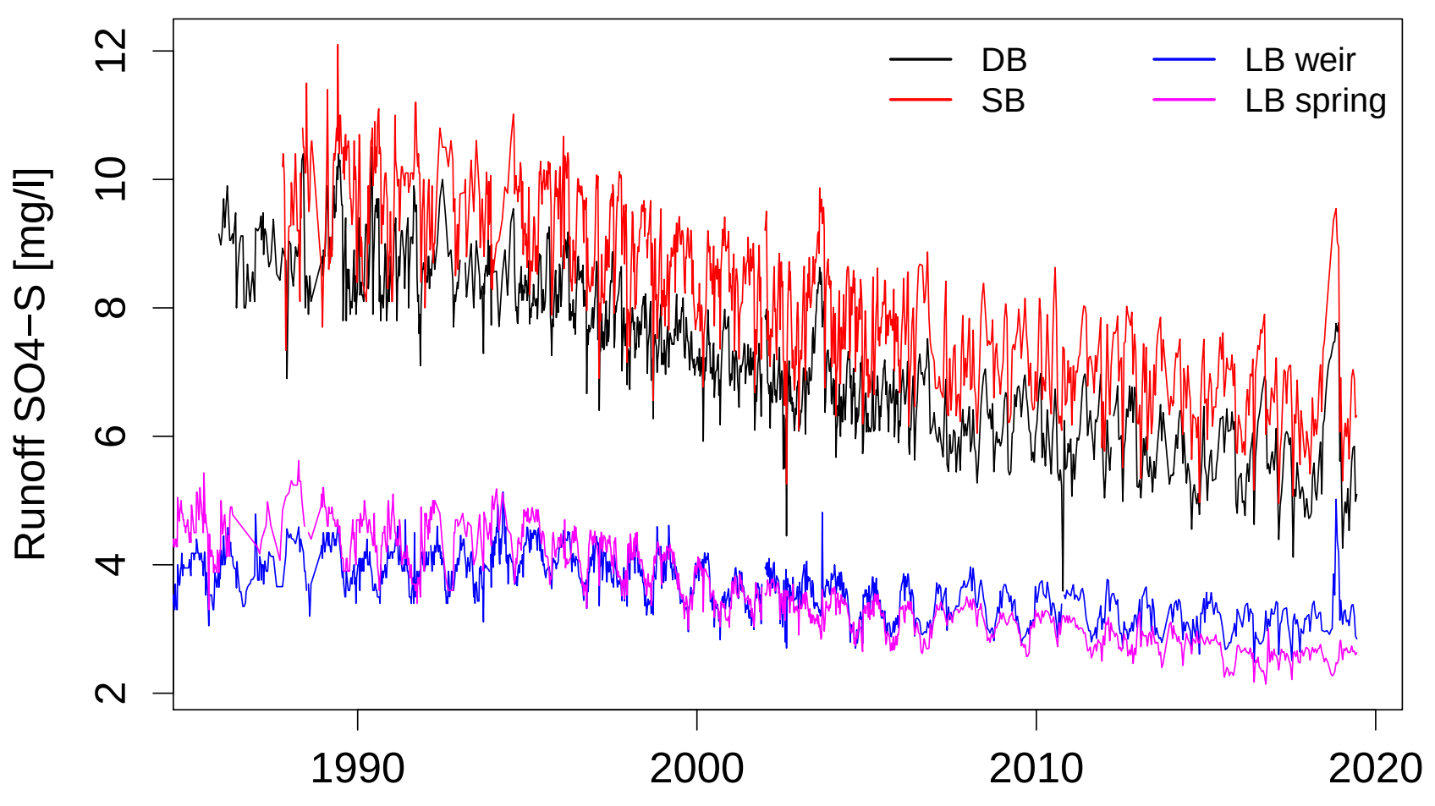


Figure 2: SO_4 in runoff at all four catchments

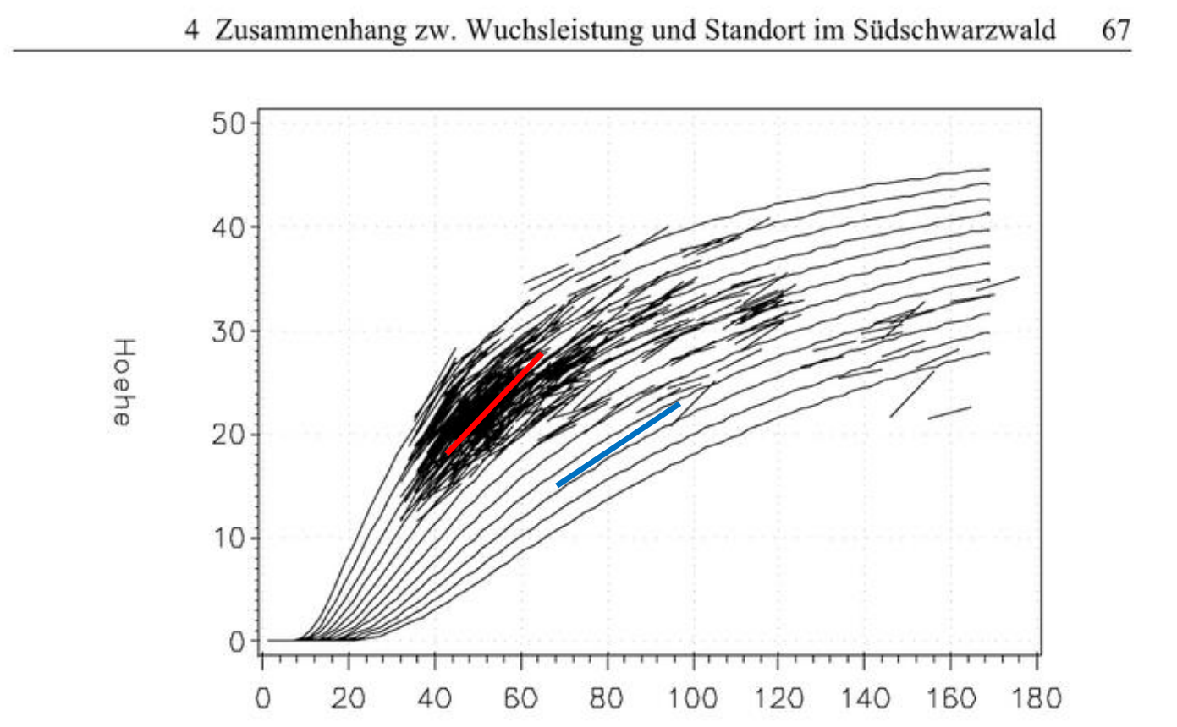


Figure 3: Height growth Norway spruce in Southern Germany. Lange Bramke in red (1988-2019) and blue (1889-1914). after: Teuffel et al. 2007

- **Modelling task example:** explain storage and transport of SO_4
- **Modelling task example:** identify processes (climate, nutrients, water) behind the change in site index

Model paradigms:

hydrology

- fluxes at boundaries (rain, runoff)
- monitor budget (mass conservation)
- runoff reflects soil states
- identify processes by inverse modelling
- abstract as algebra

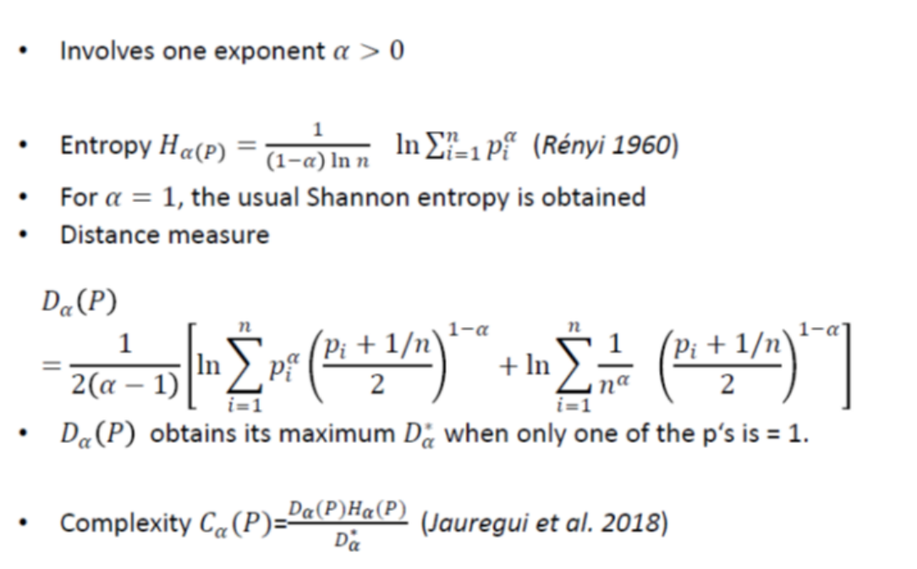
forest science

- concentrations at boundaries (moisture, nutrients)
- monitor events (damage, intervention)
- tree level: soil states represent input fluxes
- adaptation through tree-soil interaction
- abstract as coalgebra(?)

Measures of Information and Complexity

- Renyi Complexity (RC), MPR Complexity, Mean Information Gain (MIG) Permutation entropy (PE)
- reference processes: e.g. correlated noise, fBM (fractional Brownian motion)

definition



- Involves one exponent $\alpha > 0$
- Entropy $H_{\alpha(P)} = \frac{1}{(1-\alpha)\ln n} \ln \sum_{i=1}^{n} p_i^{\alpha}$ (*Rényi 1960*)
- For $\alpha = 1$, the usual Shannon entropy is obtained
- Distance measure

$$D_\alpha(P) = \frac{1}{2(\alpha-1)}\left[\ln \sum_{i=1}^{n} p_i^{\alpha}\left(\frac{p_i+1/n}{2}\right)^{1-\alpha} + \ln \sum_{i=1}^{n} \frac{1}{n^{\alpha}}\left(\frac{p_i+1/n}{2}\right)^{1-\alpha}\right]$$

- $D_\alpha(P)$ obtains its maximum D_α^* when only one of the p's is = 1.
- Complexity $C_\alpha(P) = \frac{D_\alpha(P) H_\alpha(P)}{D_\alpha^*}$ (*Jauregui et al. 2018*)

Figure 4: Ribeiro, H. V., M. Jauregui, L. Zunino and E. K. Lenzi (2017). Characterizing time series via complexity-entropy curves. Physical Review E 95(6): 062106

deterministic

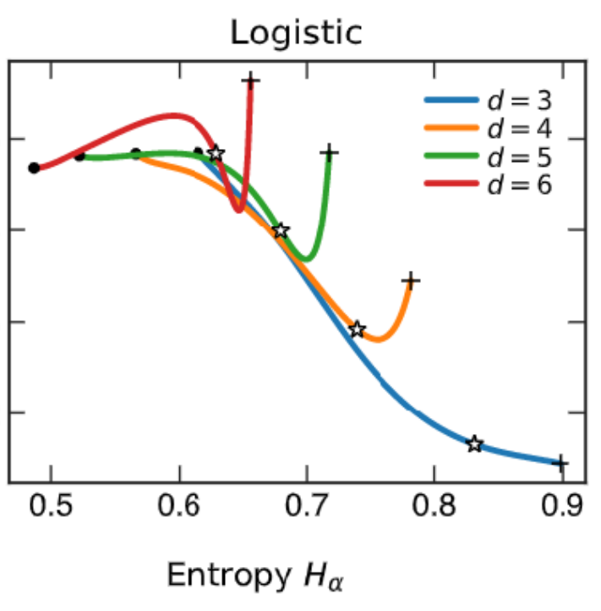


Figure 5: d is embedding dimension (window length), from: Ribeiro et al. 2017

stochastic

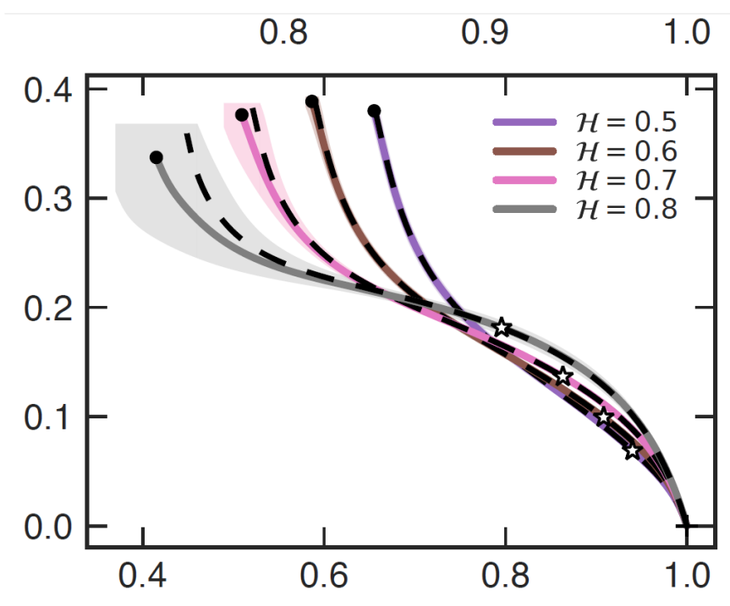


Figure 6: H is Hurst coefficient, from: Ribeiro et al. 2017

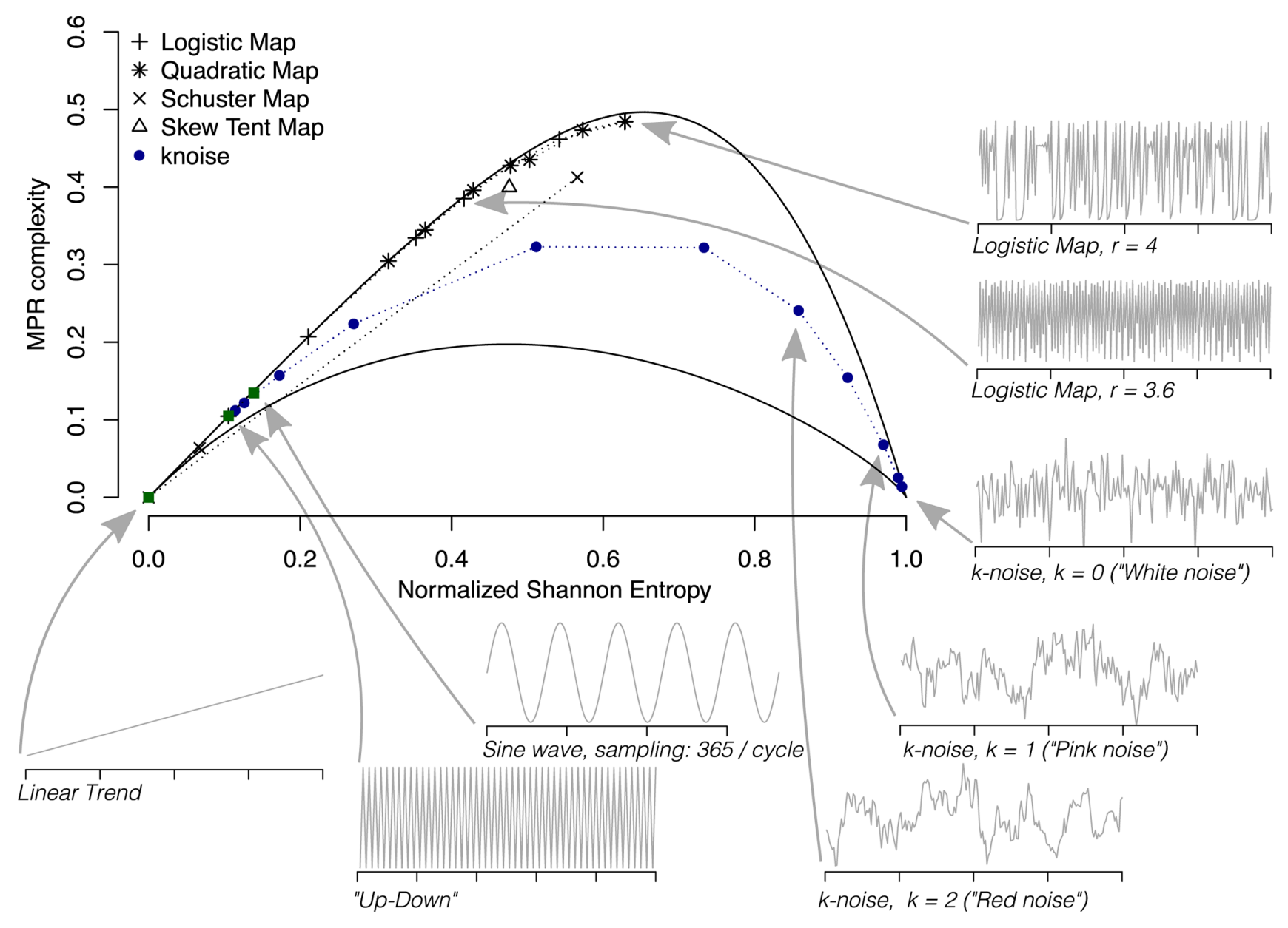


Figure 7: PE and MPR for different reference processes. from: Sippel, Sebastian, et al. "Diagnosing the dynamics of observed and simulated ecosystem gross primary productivity with time causal information theory quantifiers". PloS one 11.10 (2016): e0164960.

Results

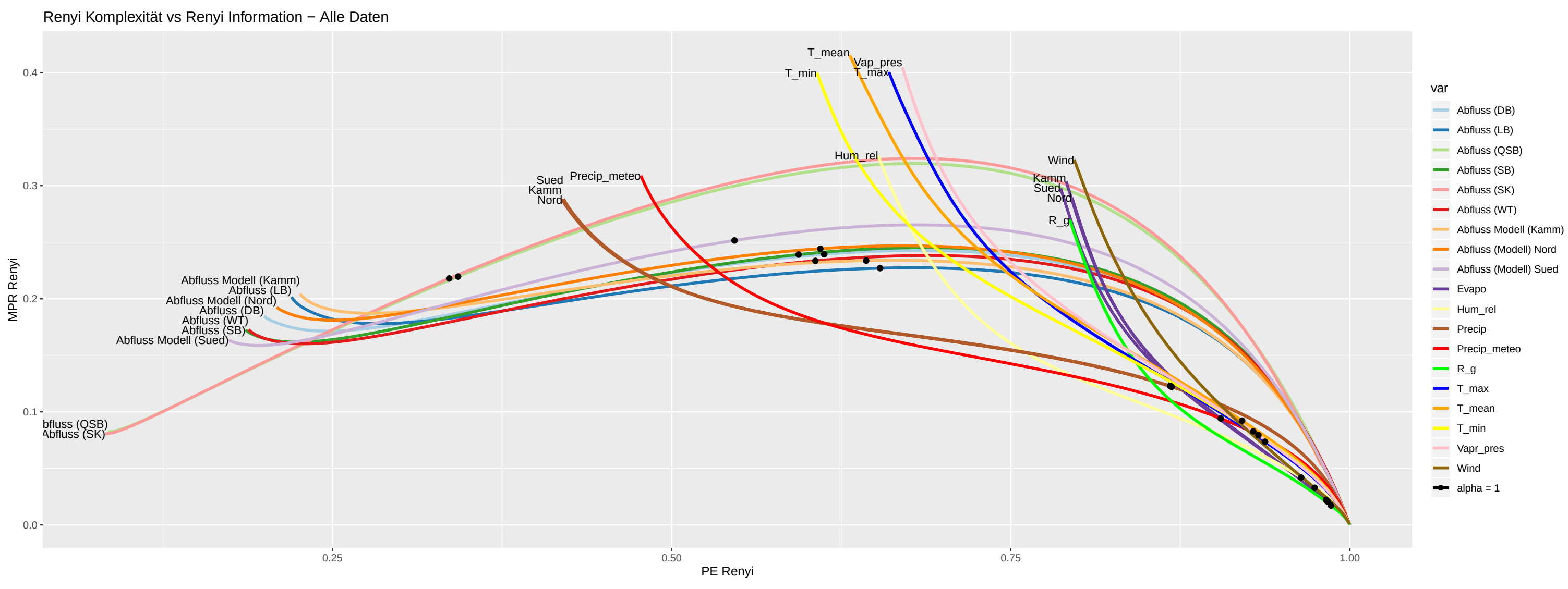


Figure 8: Renyi complexity and information for measured and modelled data of input and output variables

Conclusions

- established monitoring resolution (daily) close to maximum complexity for runoff
- input data have highest information content (randomness, entropy)

Results

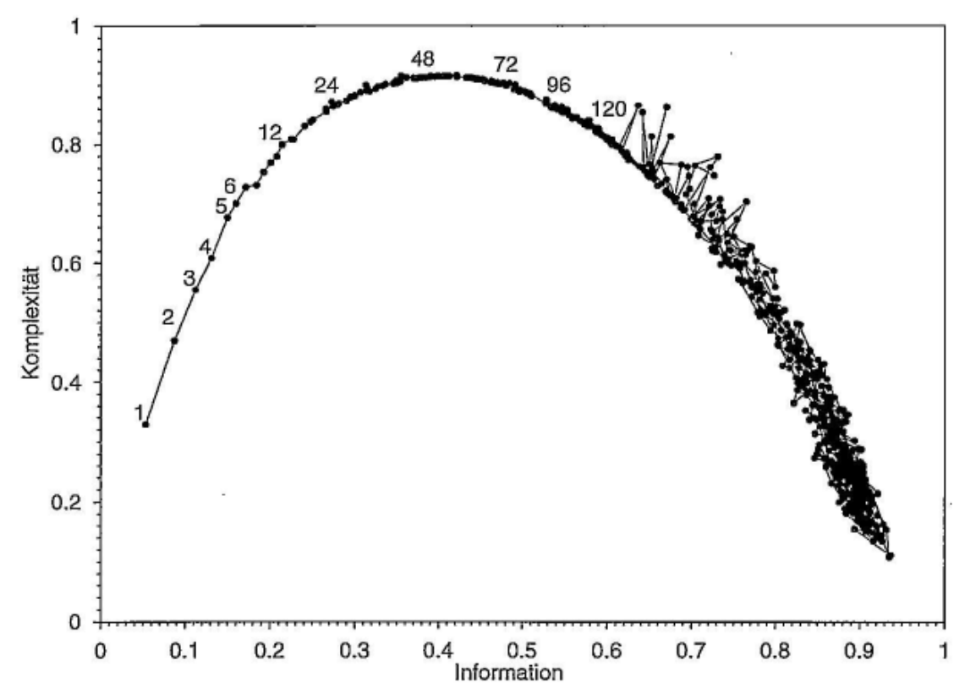


Figure 9: Complexity (FC) and Information (MIG) for runoff Lange Bramke weir at different levels of temporal aggregation (numbers give intervals in hours)

Conclusions

- model approaches in hydrology should take the growth strategies of trees into account
- information in hydrology is in the observer, but trees are in the catchment, information may thus become physical

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