Karlsruhe Institute of Technology

Related paper:
Ehret and Dej (2023)

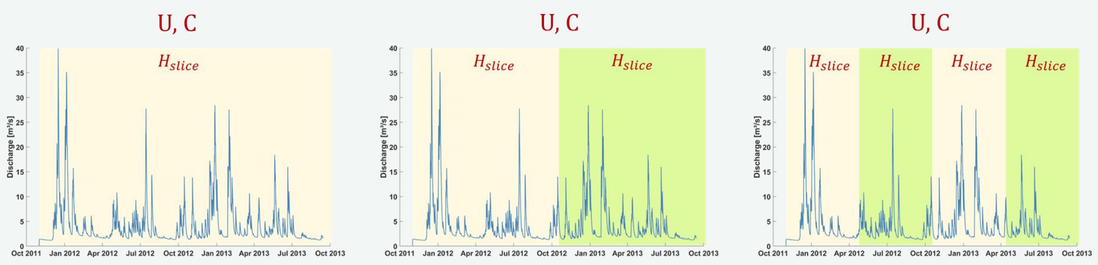
c-u-curve: A method to analyse, classify and compare dynamical systems by uncertainty and complexity

Uwe Ehret[1], Pankaj Dey[2], Sanika Baste[1]

[1] Institute of Water Resources and River Basin Management, KIT Karlsruhe, Germany, [2] Department of Hydrology, IIT Roorkee, India

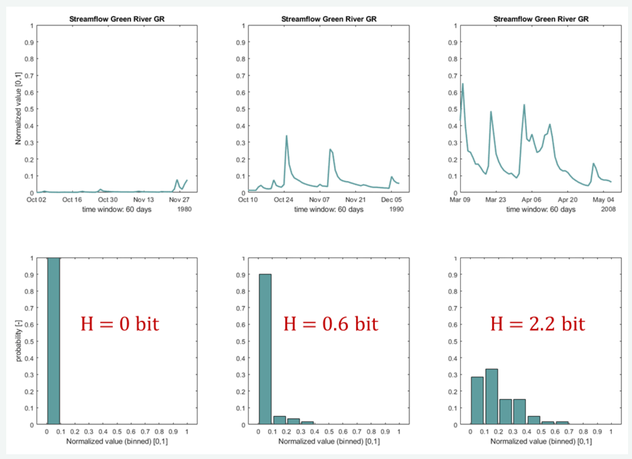
Motivation and Approach

- Characterize dynamical systems by 2 key characteristics: uncertainty and complexity
- Inspired by Hauhs and Lange (2008)



- Entropy: A general measure of variability

$$H(X) = -\sum_{vb=1}^{nvb} p(x_{vb}) \cdot \log_2(p(x_{vb}))$$



- Uncertainty: mean of all time-slice entropies

$$Uncertainty = E\big(H(X)\big) = \overline{H(X)} = \frac{1}{ns} \cdot \sum_{s=1}^{ns} H_s(X)$$

- Complexity: entropy of all time-slice entropies
"uncertainty about uncertainty"

$$Complexity = H\big(H(X)\big) = -\sum_{eb=1}^{neb} p(H_{eb}) \cdot \log_2(p(H_{eb}))$$

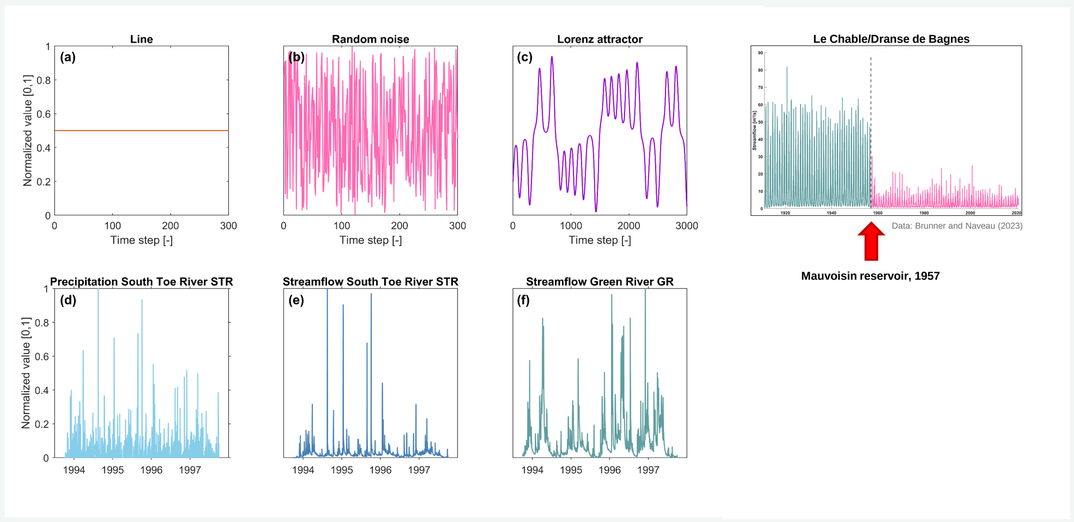
- Calculating C-U-pairs for many time-slice widths yields the "c-u-curve"

Properties

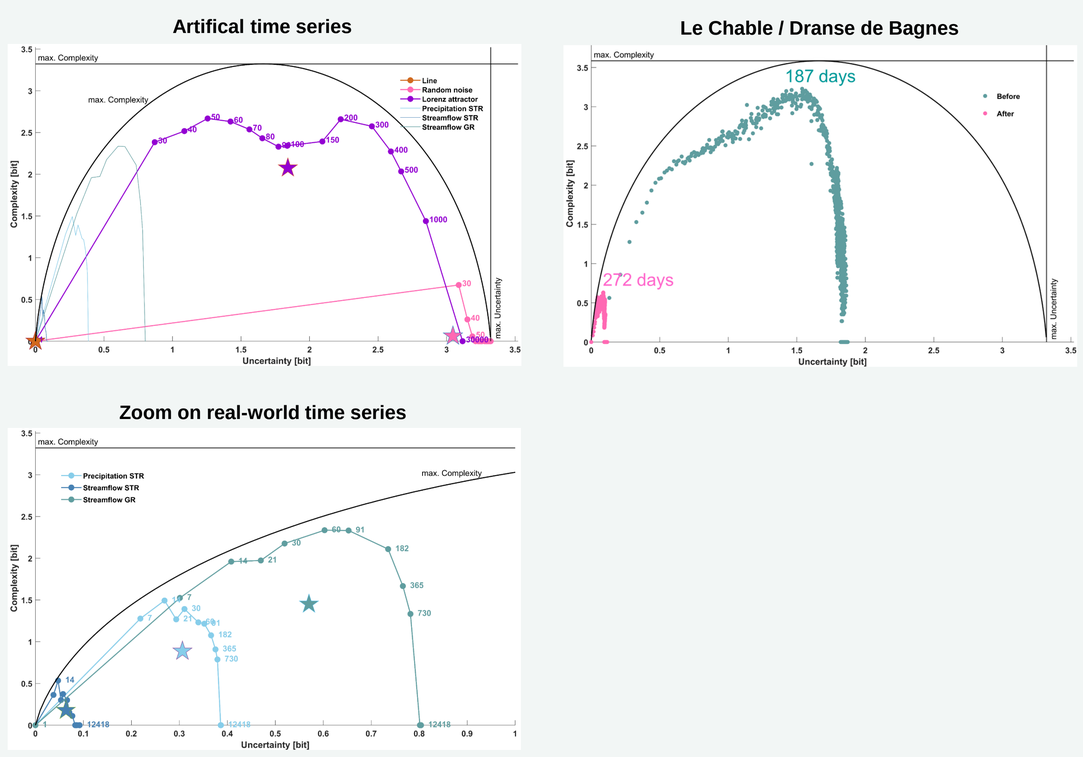
- Bounded in U and C
- Uni- and any-variate, unit-independent
- Deterministic and probabilistic
- Key statisticis: mean and max U, C, slice width at max C
- Similar but different from C_{LMC} López-Ruiz (1995) and MSE by Costa et al. (2002)

Application

- Artifical and real-world time-series



- C-u-curves



Next steps

- Normalize with max U and max C
- Find better appraoch to determine "robust region" (slice widths with sufficient population of both U and C histograms
- Analyse input – states – output transformation of dynamical systems

References

Hauhs, M., and Lange, H. (2008): Classification of Runoff in Headwater Catchments: A Physical Problem?, Geography Compass, 2, 235-254

Brunner, M. I., and Naveau, P. (2023): Spatial variability in Alpine reservoir regulation: deriving reservoir operations from streamflow using generalized additive models. HESS 27, 673–687

Ehret, U., and P. Dey (2023): Technical note: Complexity–uncertainty curve (c-u-curve) – a method to analyse, classify and compare dynamical systems, Hydrol. Earth Syst. Sci., 27(14), 2591-2605.

LopezRuiz, R., Mancini, H. L., and Calbet, X.: A statistical measure of complexity, Physics Letters A, 209, 321-326, 1995.

Costa, M., Goldberger, A. L., and Peng, C. K.: Multiscale Entropy Analysis of Complex Physiologic Time Series, Physical Review Letters, 89, 068102, 10.1103/PhysRevLett.89.068102, 2002.