Minimum Description Length, E-Values and Evidence: a brief introduction

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Abstract

- E is for Evidence -

How much evidence do the data give us about one hypothesis versus another? A standard way to measure evidence is still the p-value, despite a myriad of problems surrounding it. Alternatives are likelihood ratios and Bayes factors - but these have other problems. We present the e-value, a recently popularized notion of evidence that is a far-reaching extensions of likelihood ratios that overcomes many of the issues of the standard methods. While e-values were only given a name as recently as 2019, interest in them has since exploded with papers appearing in all the top statistics journals.

E-values can be given an information-theoretic interpretation; in fact, all e-value based methods are minimum description length (MDL) methods, but not the other way around: the log of the e-value can be interpreted as the number of additional bits by which one manages to compress the data compared to the code that would be optimal if the null hypothesis were true. Here we will introduce e-values with an emphasis on their information-theoretic, MDL interpretation, and we will provide several examples of their use. We will discuss their main virtue: e-values preserve their evidential validity under optional continuation of data collection and combination of data from different sources, and allow for standard statistical Type-I error control in such scenarios. We will discuss why the codes originally used in MDL do not always have this highly important property.

References/Recommended reading

P. Grunwald,. R. de Heide, W. Koolen (2023): Safe Testing. To appear in J. Roy. Stat. Soc., Series B. Freely available on https://doi.org/10.48550/arXiv.1906.07801. The first two sections of this paper are a good preparation.

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