Information Theory for Hypothesis Testing

2020 Summer School for Information Theory in the Earth Sciences (SITES)

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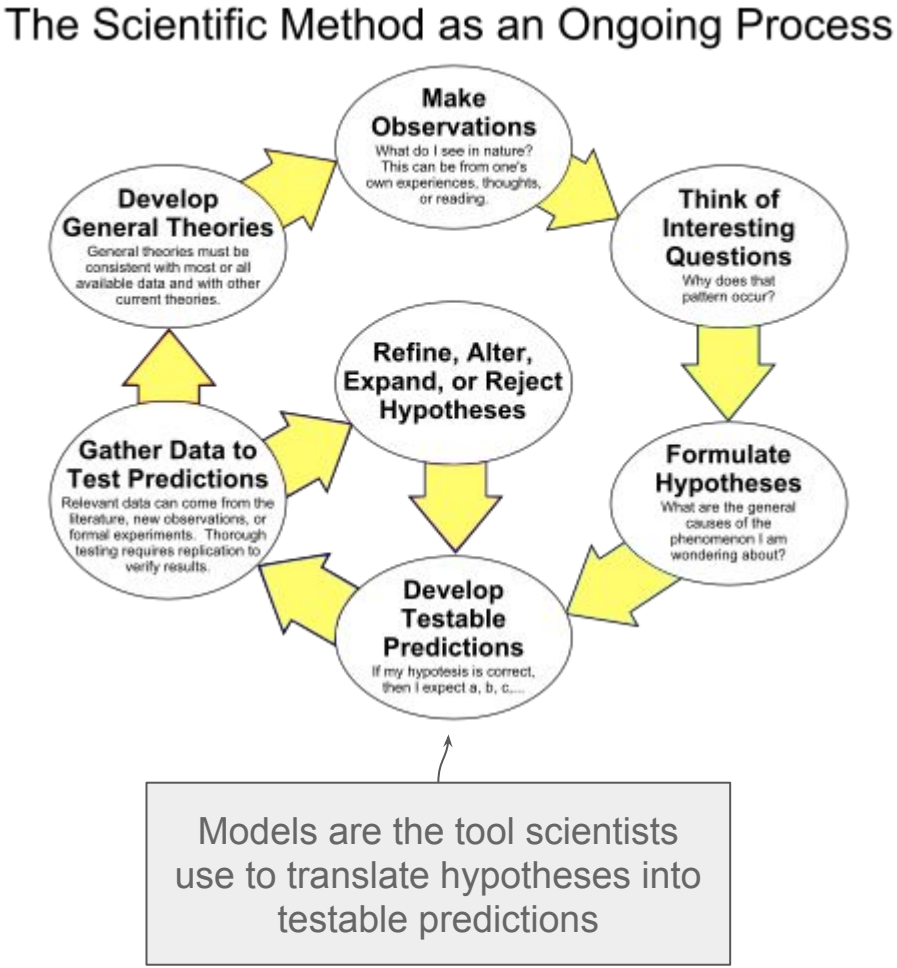
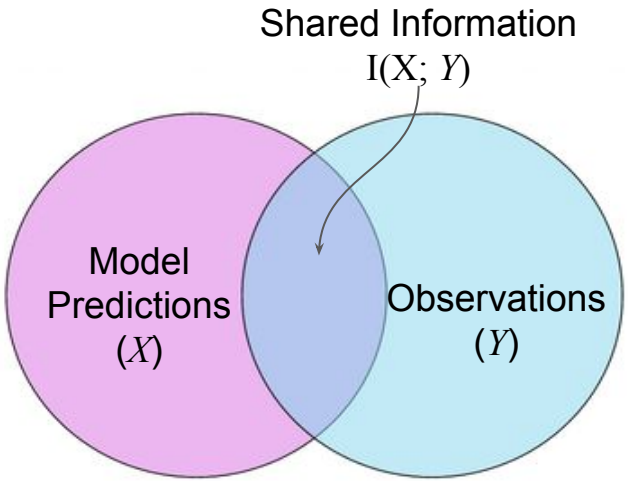
Models as Hypotheses

As scientists, we evaluate models to

- Assess adequacy for a particular task (applied science)
- Test a hypothesis (basic science)
- Others … ?

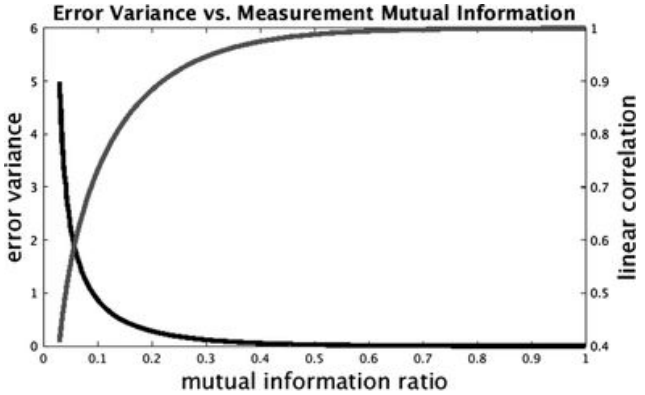


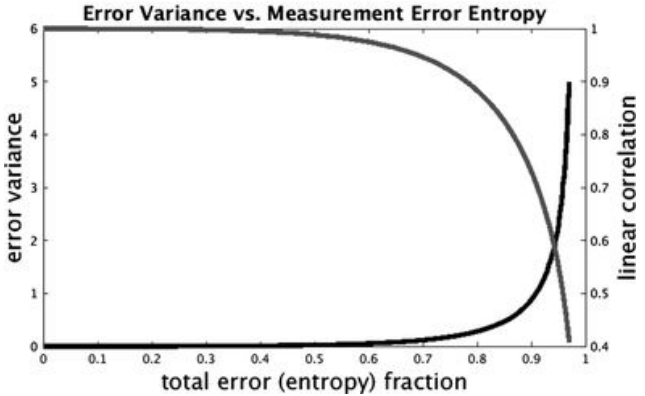
Mutual Information as an Evaluation Metric



- Nonlinear
- Nonparametric
- Sensitive to differences between ‘good’ models

$$Y = X + \varepsilon; \quad \varepsilon \sim N[0, \sigma]$$





Information Theory Hypothesis Testing

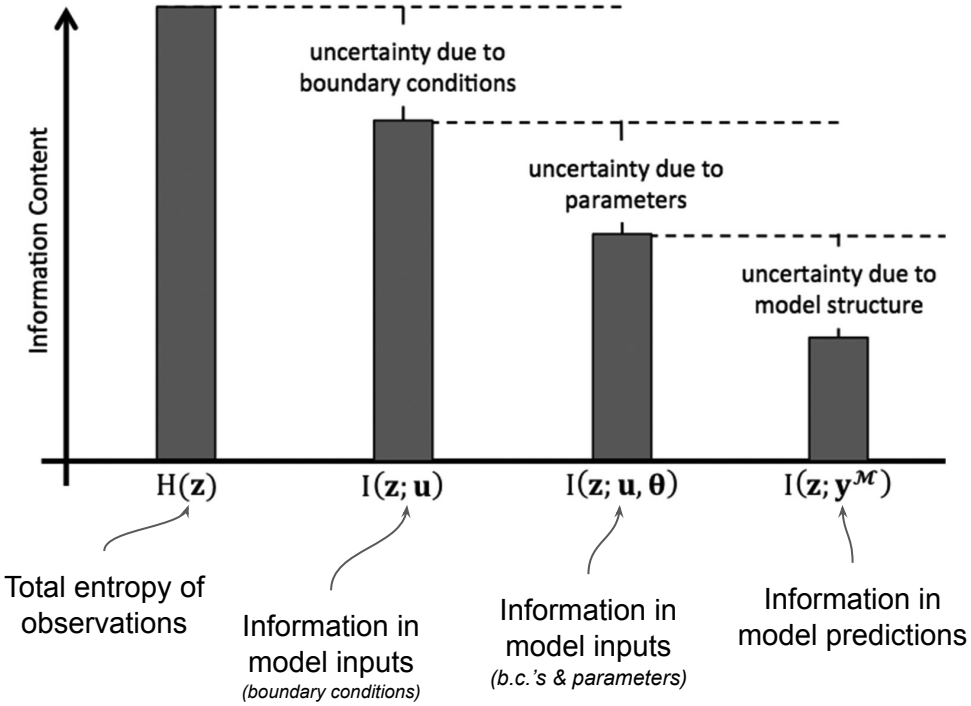
What is hypothesis testing?

Are hypotheses true or false?

What does it mean to assign a probability to a hypothesis?

Proposal: What we really want to measure when we test a hypothesis is the amount of information that hypothesis provides.

The model is: $z = f(u, \theta)$

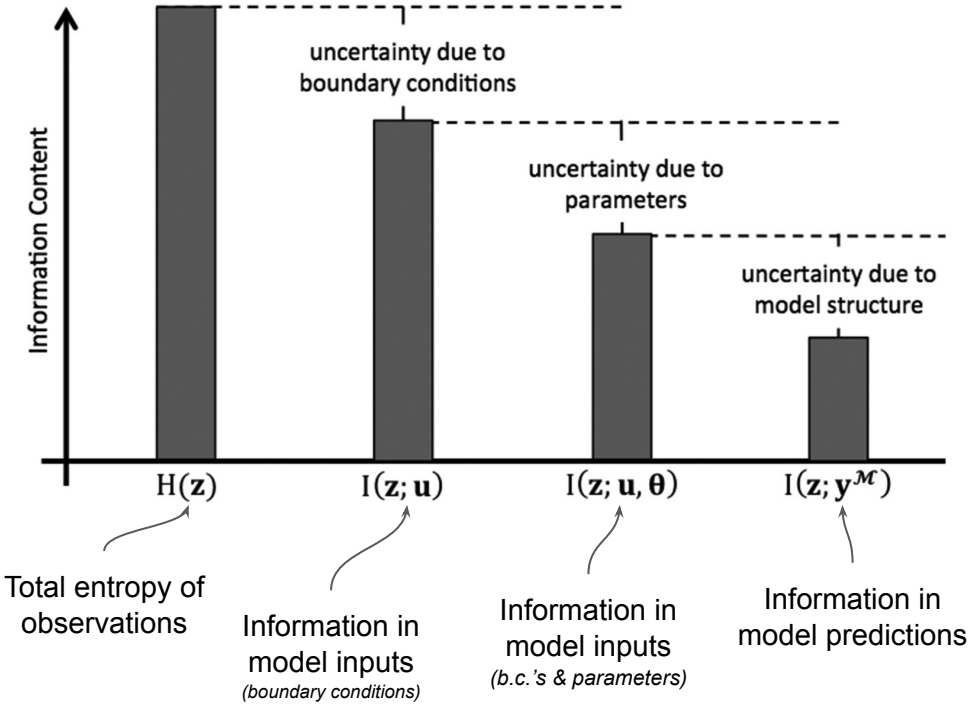


Information Theory Hypothesis Testing

Problem: Cannot measure information shared between observations (u, y).

Solution: Approximate by using a data-driven model. This is as close as we can get to asking how much information is in the data themselves, independent of any conceptual hypothesis.

The model is: $z = f(u, \theta)$



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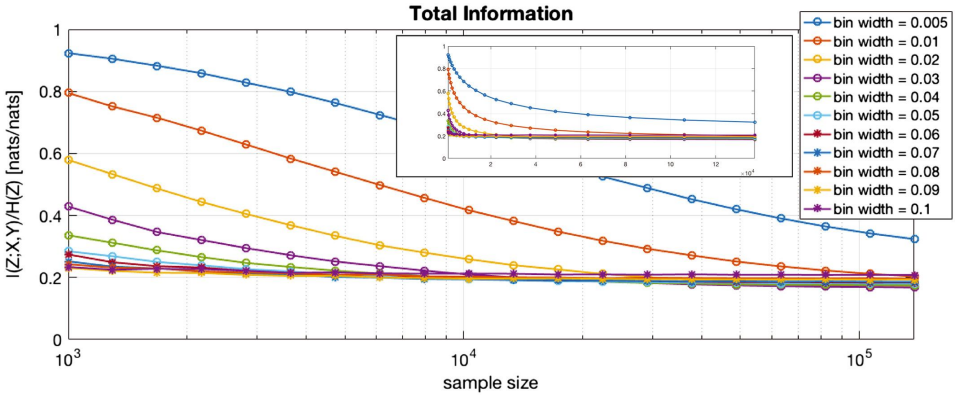

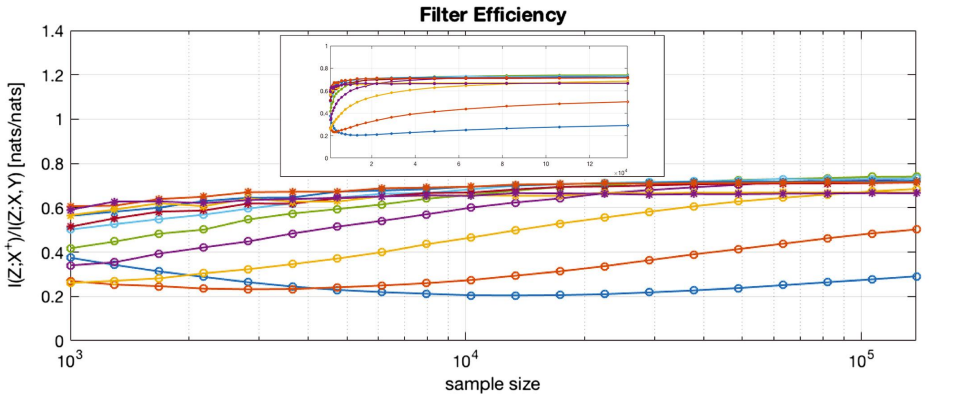

There are several ways to calculate mutual information, which all have slightly different characteristics.

A histogram (binning) method effectively discretizes the data at a specified resolution, and lets you test models/hypotheses at a particular level of precision.

The maximum level of precision depends on quantity of available data.