Maximum Entropy Production Model of Heat Fluxes – An Application of Information Theory in Modeling Earth System

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- **Information Entropy**

  (Shannon, 1948)

$$S_I = -\sum_{i=1}^{n} p_i \ln p_i$$

- **Maximum Entropy (MaxEnt) Principle**
  (Jaynes, 1957)

$$p_i = \frac{1}{Z}\exp[-\sum_{i=1}^{n}\vec{\lambda}\cdot\vec{f}(i)]$$

$$S_I^{Max} = \max_{\{p_i\}}\{S_I \mid \sum_{i=1}^{n} p_i \vec{f}(i) = \vec{F}\}$$

$$= \ln Z(\vec{\lambda}) - \vec{\lambda}\cdot\vec{F}$$

- **MaxEnt to Equilibrium Thermodynamics** (Tribus, 1961)

$$S = \kappa S_I^{\max}$$

$$TdS = \delta Q = dU + PdV$$

- **MaxEnt to Non-equilibrium Thermodynamics - Maximum Entropy Production Principle (MEP/MaxEP)**

(Dewar, 2003, 2005, 2014)

Irreversibility

$$I(\vec{F}) \equiv \int p(\vec{f}) \ln \frac{p(\vec{f})}{p(-\vec{f})} d\vec{f} = 2\vec{\lambda}(\vec{F}) \cdot \vec{F}$$

where $\vec{F} = \int p(\vec{f})\vec{f}d\vec{f}$ - macroscopic fluxes

"MaxEnt is equivalent to an extremal selection criterion of (macroscopic fluxes) MEP"

$$\vec{F}(C) = \max_{\vec{F}|C}/\min \left\{ I(\vec{F}) \right\}$$

C: macroscopic physical constraints on the fluxes

The irreversibility $I(\vec{F})$ is equivalent to

- physical variables thermodynamic entropy production and mechanical energy dissipation (*Dewar and Maritan, 2014*) and MEP as a physical principle;

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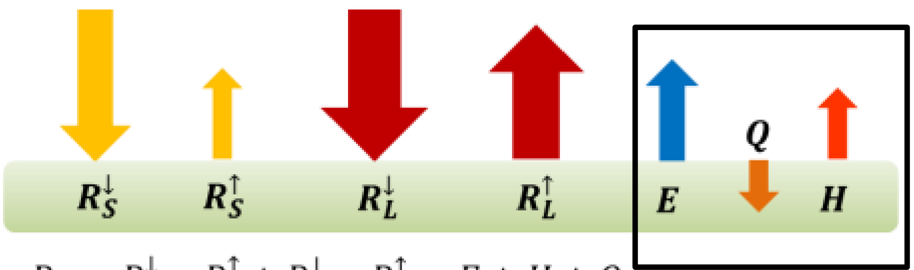
- physical variables thermodynamic entropy production and mechanical energy dissipation (*Dewar and Maritan, 2014*) and “MEP” viewed as a physical principle;
- non-physical variables (*Wang and Bras, 2011*) and “MEP” treated as an inference algorithm.

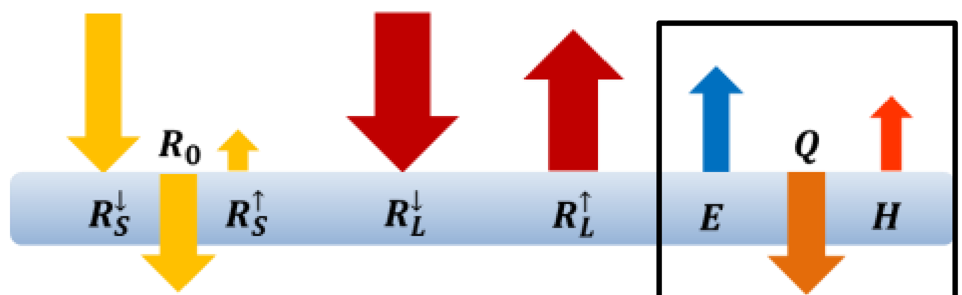
Surface Energy Budgets

Land



$$R_n \equiv R_S^{\downarrow} - R_S^{\uparrow} + R_L^{\downarrow} - R_L^{\uparrow} = E + H + Q$$

Ocean



$$R_n^S \equiv R_S^{\downarrow} - R_S^{\uparrow} = R_0$$
$$R_n^L \equiv R_L^{\downarrow} - R_L^{\uparrow} = E + H + Q$$

$$\vec{F} = (E, H, Q)$$

$$\vec{\lambda}(\vec{F}) = \left(\frac{E}{\sigma I_0 |H|^{\frac{1}{6}}}, \frac{H}{I_0 |H|^{\frac{1}{6}}}, \frac{Q}{I_s} \right)$$

(Wang and Bras, 2009; 2011)

$$\vec{F} = (E, H, Q)$$

$$\vec{\lambda}(\vec{F}) = \left(\frac{E}{\sigma I_0 |H|^{\frac{1}{6}}}, \frac{H}{I_0 |H|^{\frac{1}{6}}}, \frac{Q}{I_s} \right)$$

$$I(\vec{F}) = \vec{\lambda}(\vec{F}) \cdot \vec{F} = \frac{E^2}{\sigma I_0 |H|^{\frac{1}{6}}} + \frac{H^2}{I_0 |H|^{\frac{1}{6}}} + \frac{Q^2}{I_s}$$

Monin-Obukhov similarity equations for the atmospheric boundary layer turbulence

(*Wang and Bras*, 2010),

$$I_0 = \rho c_p \sqrt{C_1 \kappa z} \left(C_2 \frac{\kappa z g}{\rho c_p T_0} \right)^{\frac{1}{6}},$$

$$\sigma(T_s, q_s) = \sqrt{\alpha_K} \frac{\lambda^2}{c_p R_v} \frac{q_s}{T_s^2}$$

Surface specific humidity

Surface temperature

	unstable	*stable*
C_1	$\sqrt{3}/\alpha$	$2/(1+2\alpha)$
C_2	$\gamma_2/2$	2β

α, β, γ_2 are universal empirical constants in the M-O similarity equations [*Businger et al*, 1971].

$$\vec{F}(C) = \max_{\vec{F}|C}/\min\left\{I(\vec{F})\right\}$$

C: surface energy balance $E + H + Q = \begin{cases} R_n & \text{land} \\ R_n^L & \text{water} \end{cases}$

Max: maximal irreversibility or farthest to equilibrium

Min: minimal irreversibility or closest to equilibrium

MEP Model for land Surface

$$Q = \frac{B(\sigma)}{\sigma}\frac{I_s}{I_0}\left|H\right|^{-\frac{1}{6}}H$$

$$E = B(\sigma)H$$

$$E + H + Q = R_n$$

$$B(\sigma) = 6\left(\sqrt{1 + \frac{11}{36}\sigma} - 1\right),$$

$$\sigma = \sqrt{\alpha_K}\frac{\lambda^2}{c_p R_v}\frac{q_s}{T_s^2}$$

(*Wang and Bras, 2011*)

MEP Model of Transpiration

$$E = \frac{R_n}{1 + B^{-1}(\sigma)}$$

$$H = \frac{R_n}{1 + B(\sigma)}$$

MEP Model for Water-Snow-Ice Surface

$$Q = \frac{B(\sigma)}{\sigma} \frac{I_{wsi}}{I_0} |H|^{-\frac{1}{6}} - R_n^S$$

$$E = B(\sigma)H$$

$$E + H + Q = R_n^L$$

$$\sigma(T_s) = \sqrt{\alpha_K} \frac{\lambda^2}{c_p R_v} \frac{q_s^{sat}(T_s)}{T_s^2}$$

(*Wang et al, 2014*)

Bulk Transfer Models

$$H = \rho c_p C_H \left(T_s - T_a\right)$$

$$E = \rho \lambda_v C_E \left(q_s - q_a\right)$$

H, ***E*** surface turbulent latent and sensible heat flux,
T_s, T_a surface, air and soil temperature,
q_s, q_a surface and air humidity,
C_H, C_E transfer coefficients

The Project for Intercomparison of Land-surface Parameterisation Schemes (**PILPS**)

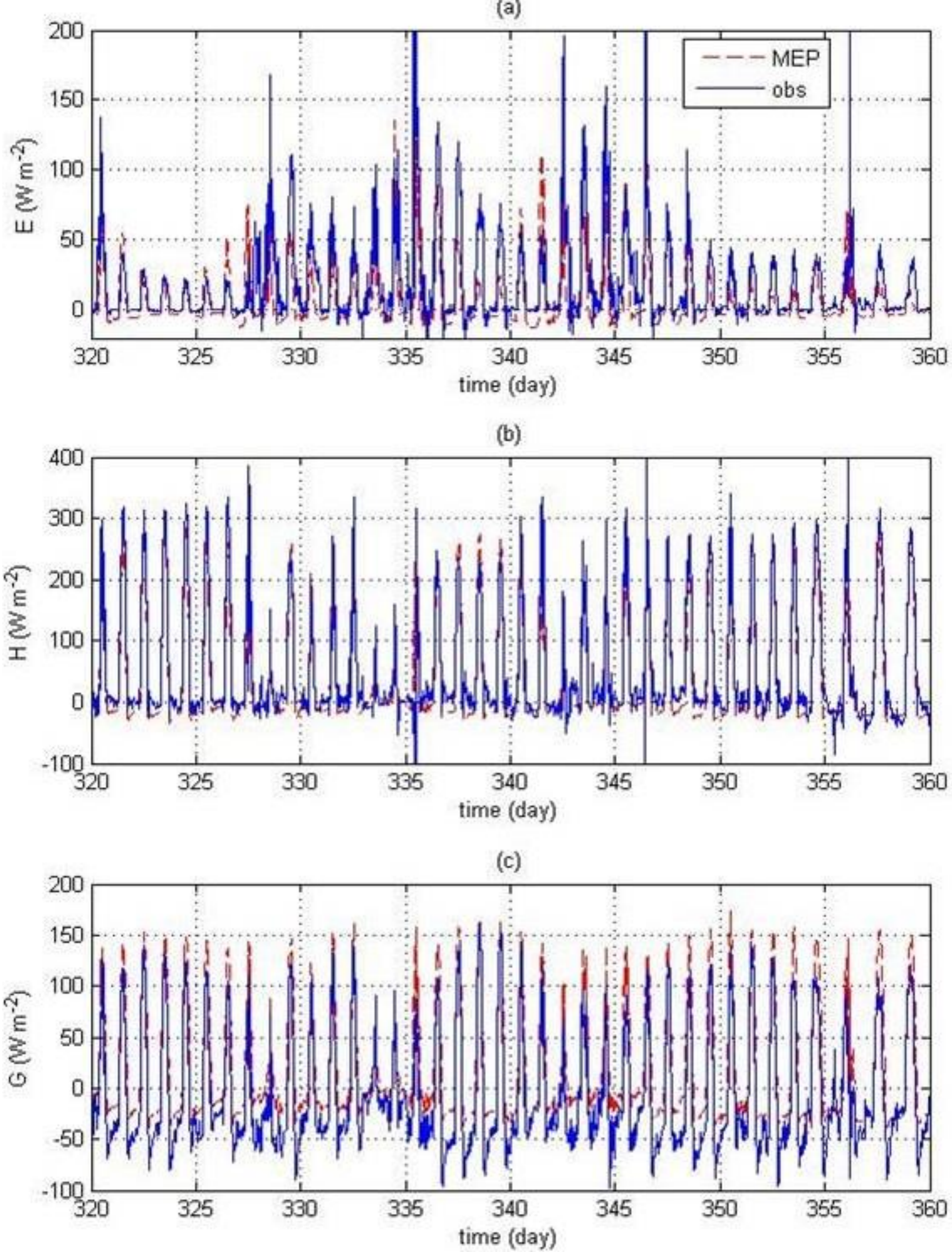
"… *no single land surface model is capable of capturing all features of the surface energy balance under all conditions* …"

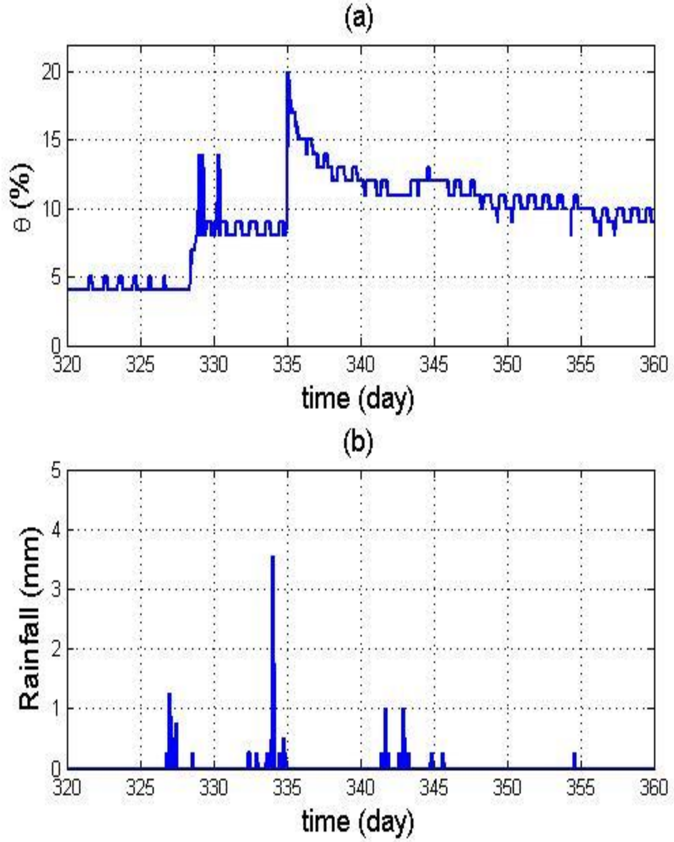
(Desborough et al., 1996; Henderson-Sellers et al., 2003)

- Lack of energy constraint,
- Measurement errors of bulk gradients,
- Uncertainties of (wind speed and roughness lengths dependent) transfer coefficients,
- Modeling errors of bulk transfer formulae (semi-empirical 1st order closure of Reynolds decomposition of turbulent flow).

	Surface Energy Budget	Input data	Model Parameters	Modeling Error
Bulk Method	Not balanced	Temperature & moisture gradient	Wind speed, roughness lengths, etc.	Semi-empirical (first order closure)
MEP Method	Balanced	Not used	Not used	First-principles (MaxEnt, MEP, MOST) bounded by radiation data

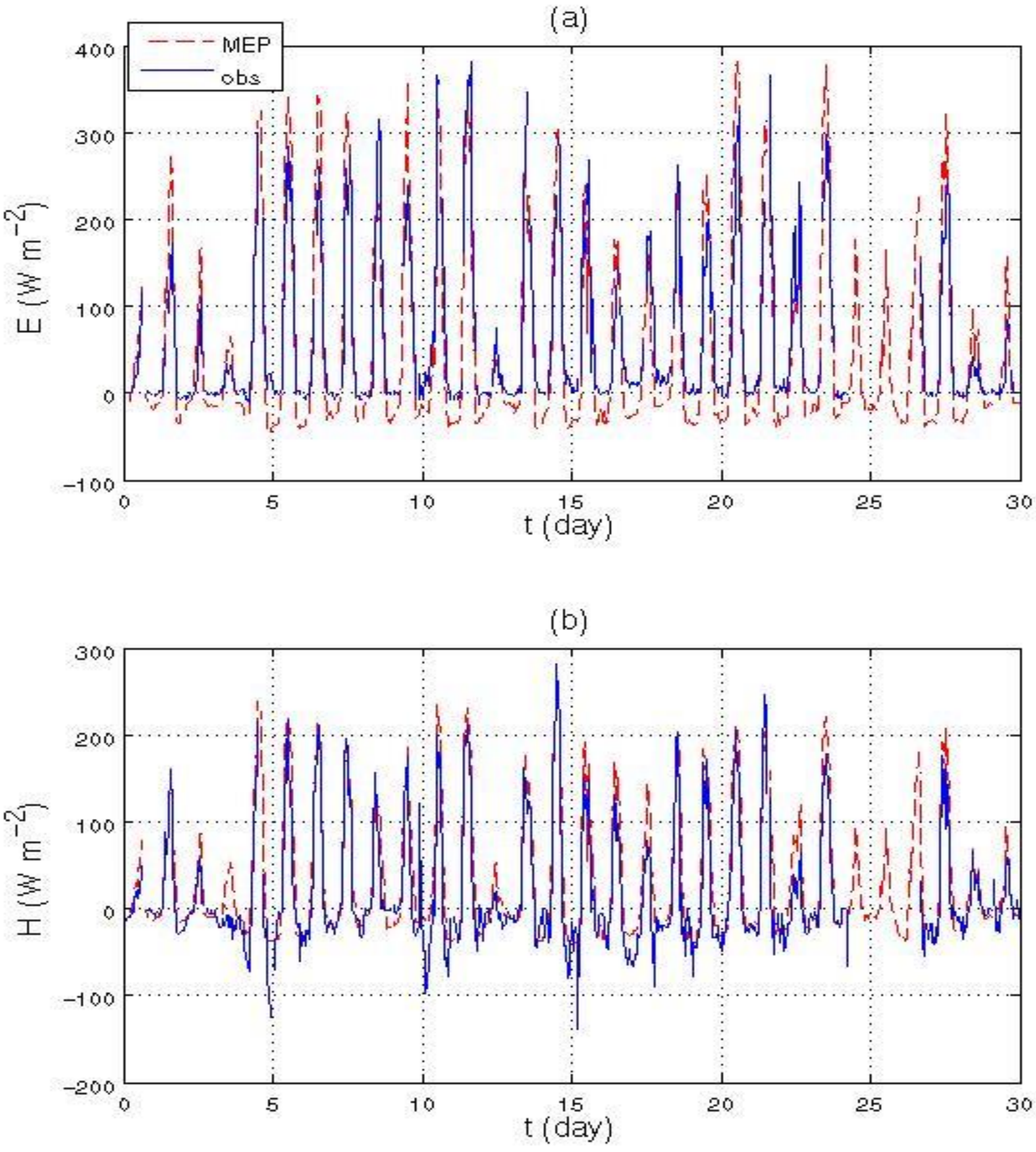
Bare Soil



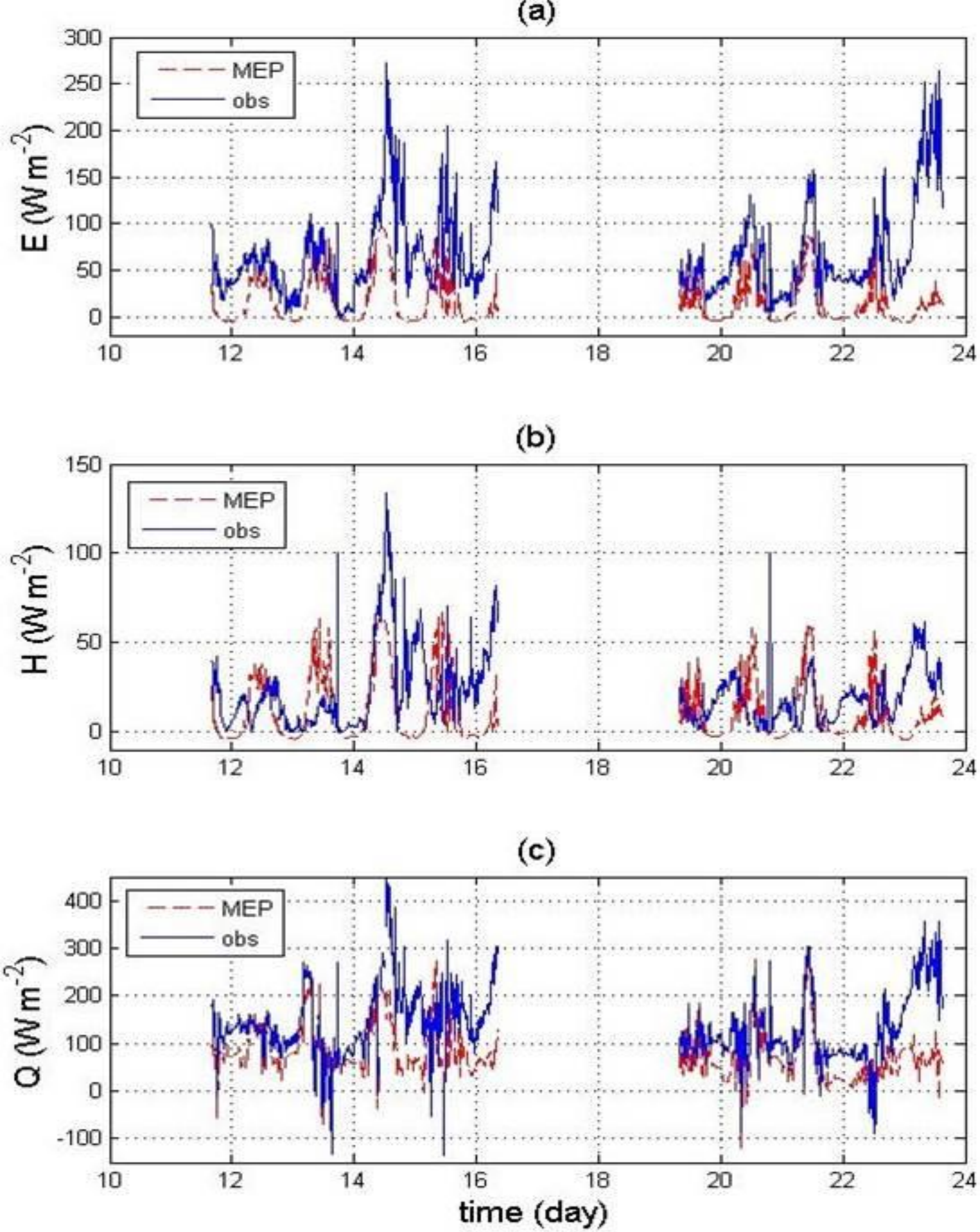


Lucky Hills site of the Walnut Gulch Experimental Watershed during 11/16 – 12/26, 2007.

Canopy

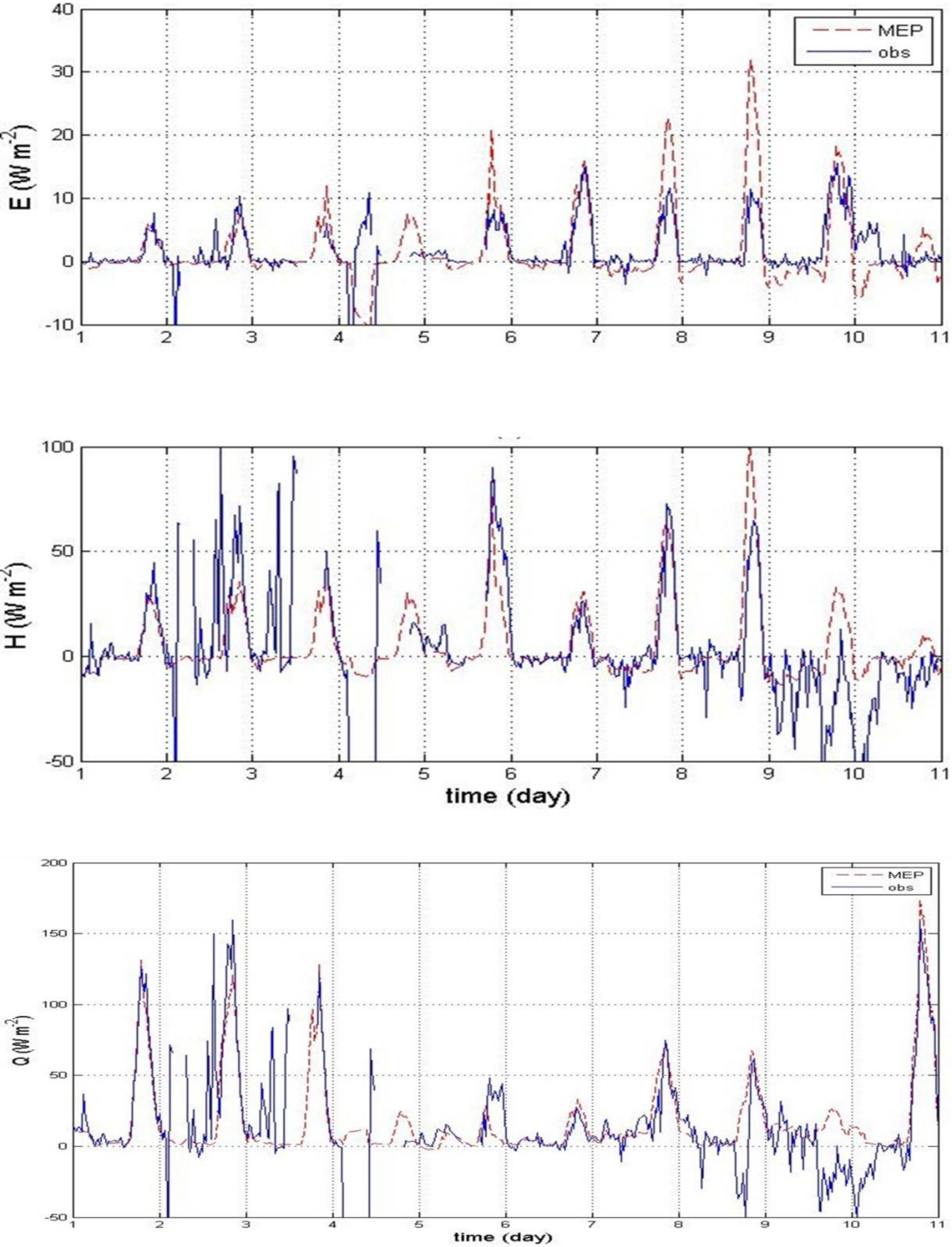


Harvard Forest site during 19 August - 18 September 1994.



Water

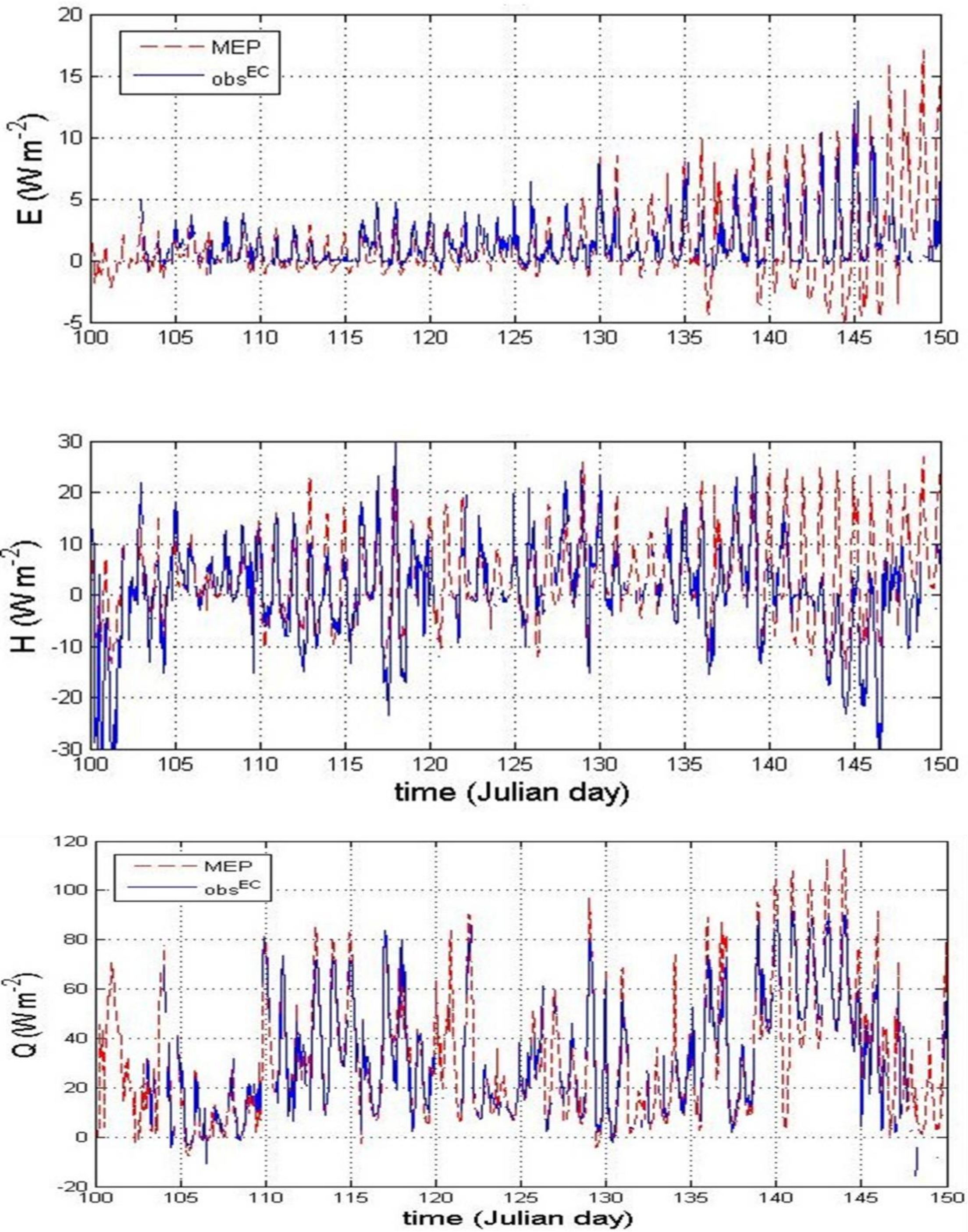
NOrthern hemisphere climate Processes field Experiment (NOPEX) that made eddy-covariance measurements turbulent fluxes over Lake Raksjo (1.5 km^2 in surface area and 4m in depth) during June and July of 1994. (Data courtesy of Sven Halldin).



Snow

FLUXNET eddy-covariance data of surface fluxes and meteorological variables collected at a snow covered grassland site in Lethbridge, Alberta, Canada (AB-GRL) during 1 - 10 December 2007. Courtesy of Charmaine Hrynkiw, Betsy Sheffield, and Mary J. Saddington.

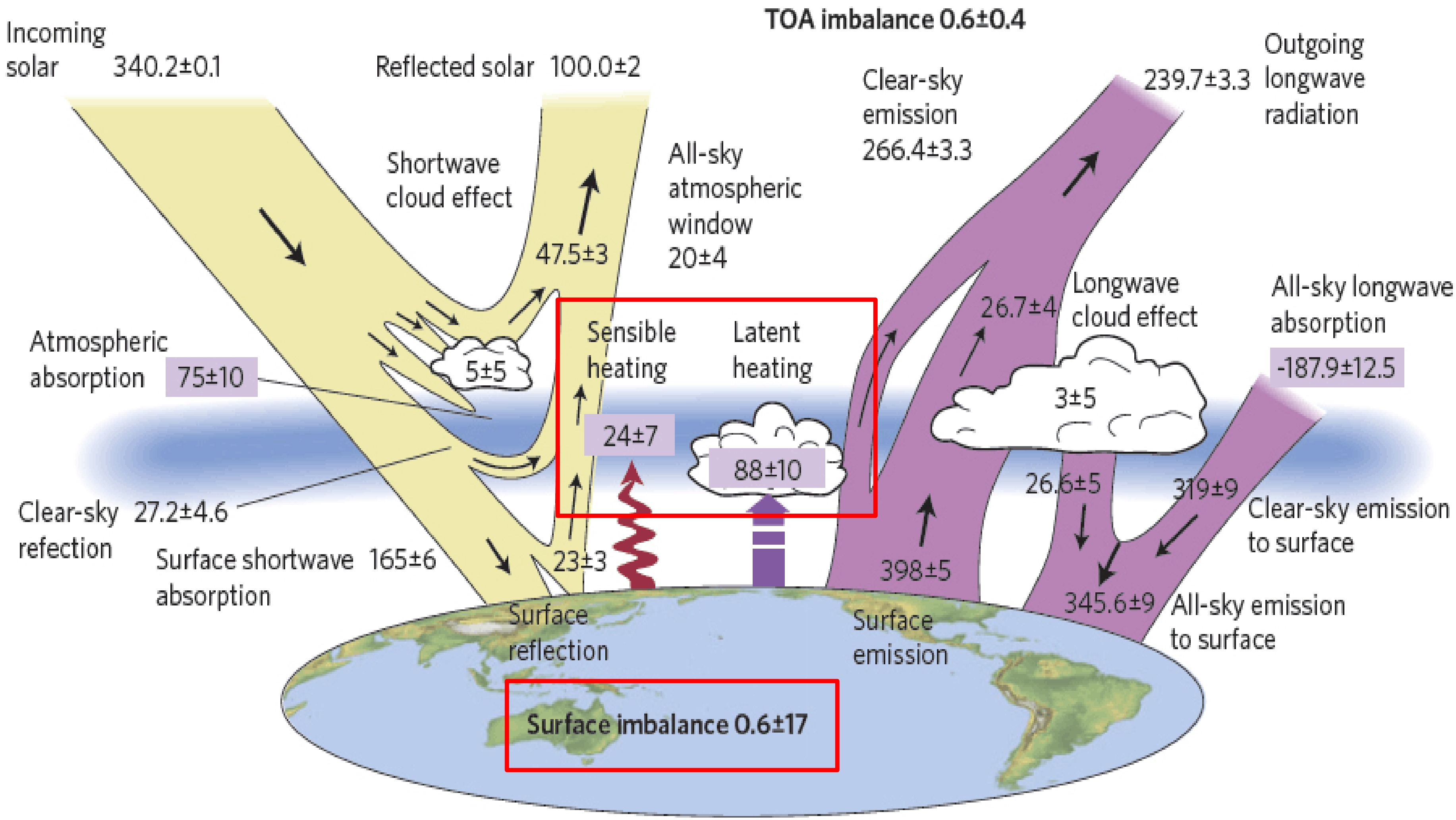
Sea Ice



Phase II eddy-covariance measurements of turbulent fluxes and meteorological variables from the Surface Heat Budget of the Arctic Ocean (SHEBA) experiment at a ice pack of the Arctic Ocean during 10 April - 30 May 1998. Courtesy of Judith A. Curry and Carol Anne Clayson.

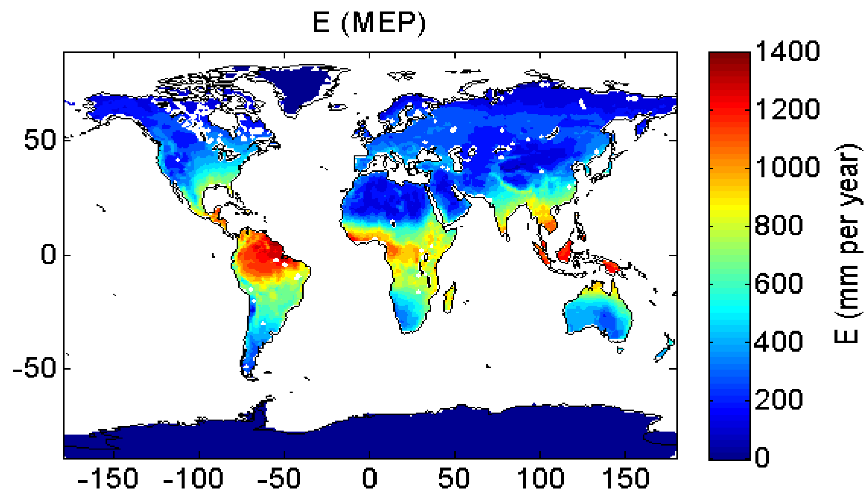
Applications of MEP Model

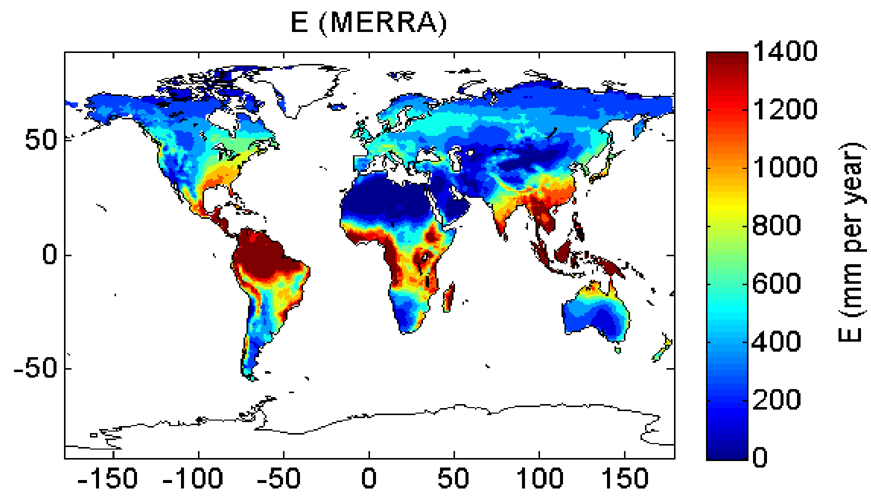
- Retrieval Algorithm of Global Surface Fluxes
- Scheme of Surface Energy Budgets in Land-Ocean-Atmospheric Models

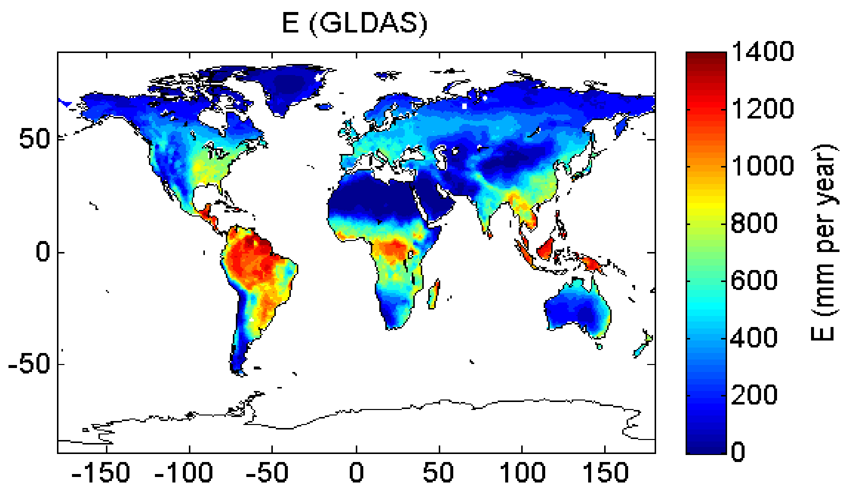


(Stephens et al., 2012)

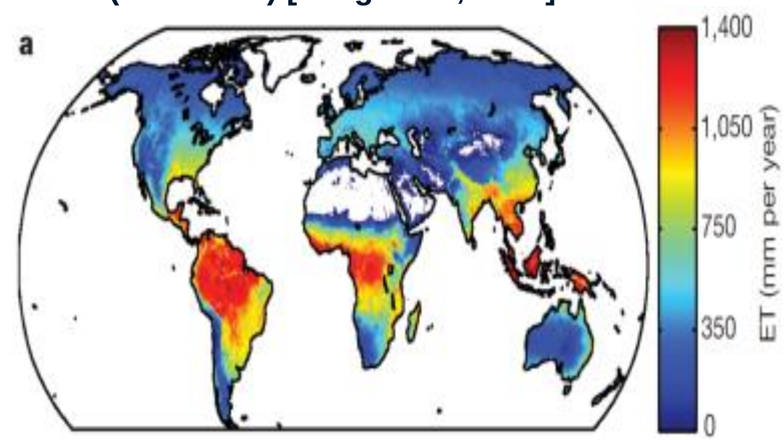
Climatology (2001-2010)



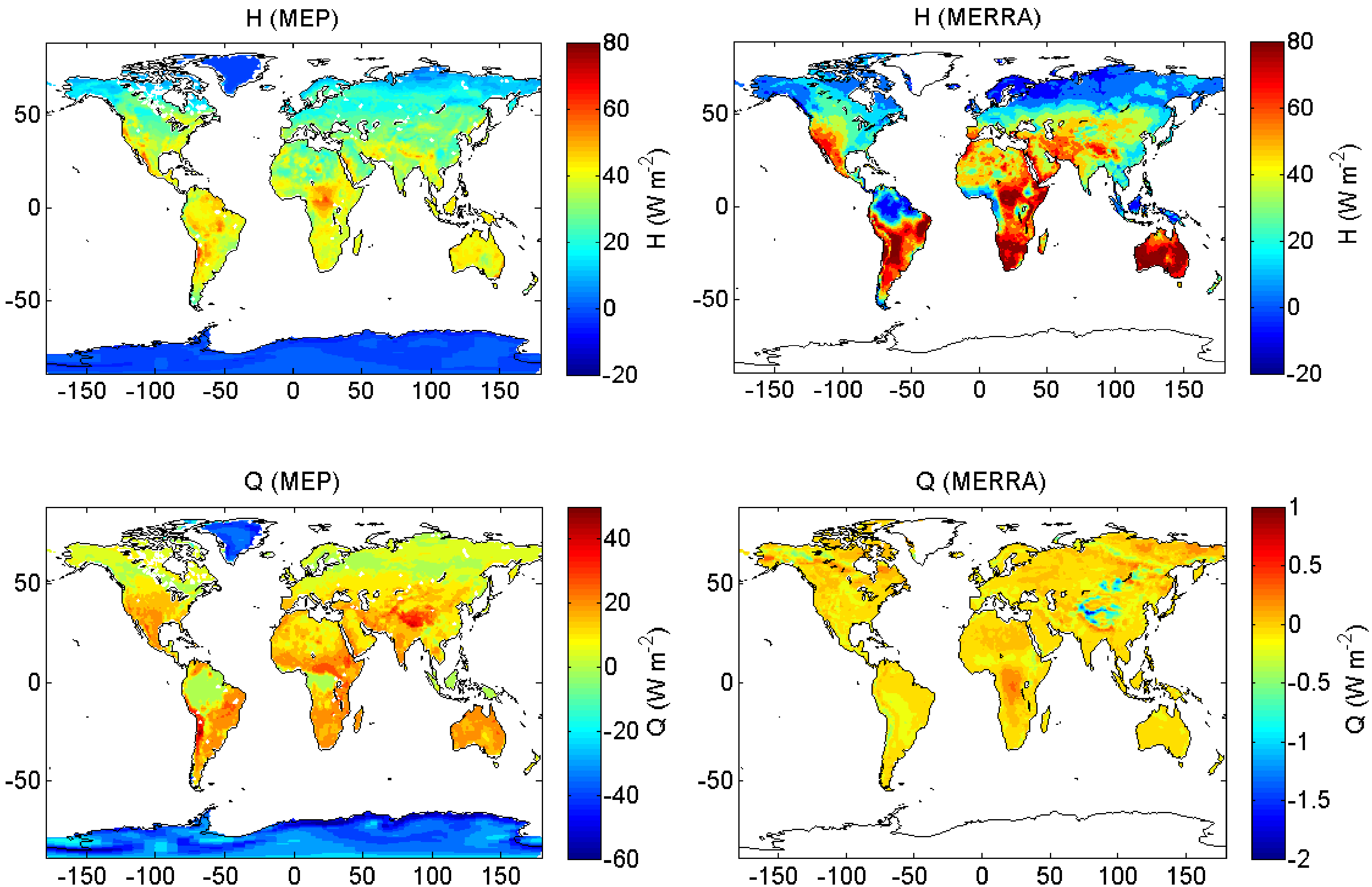




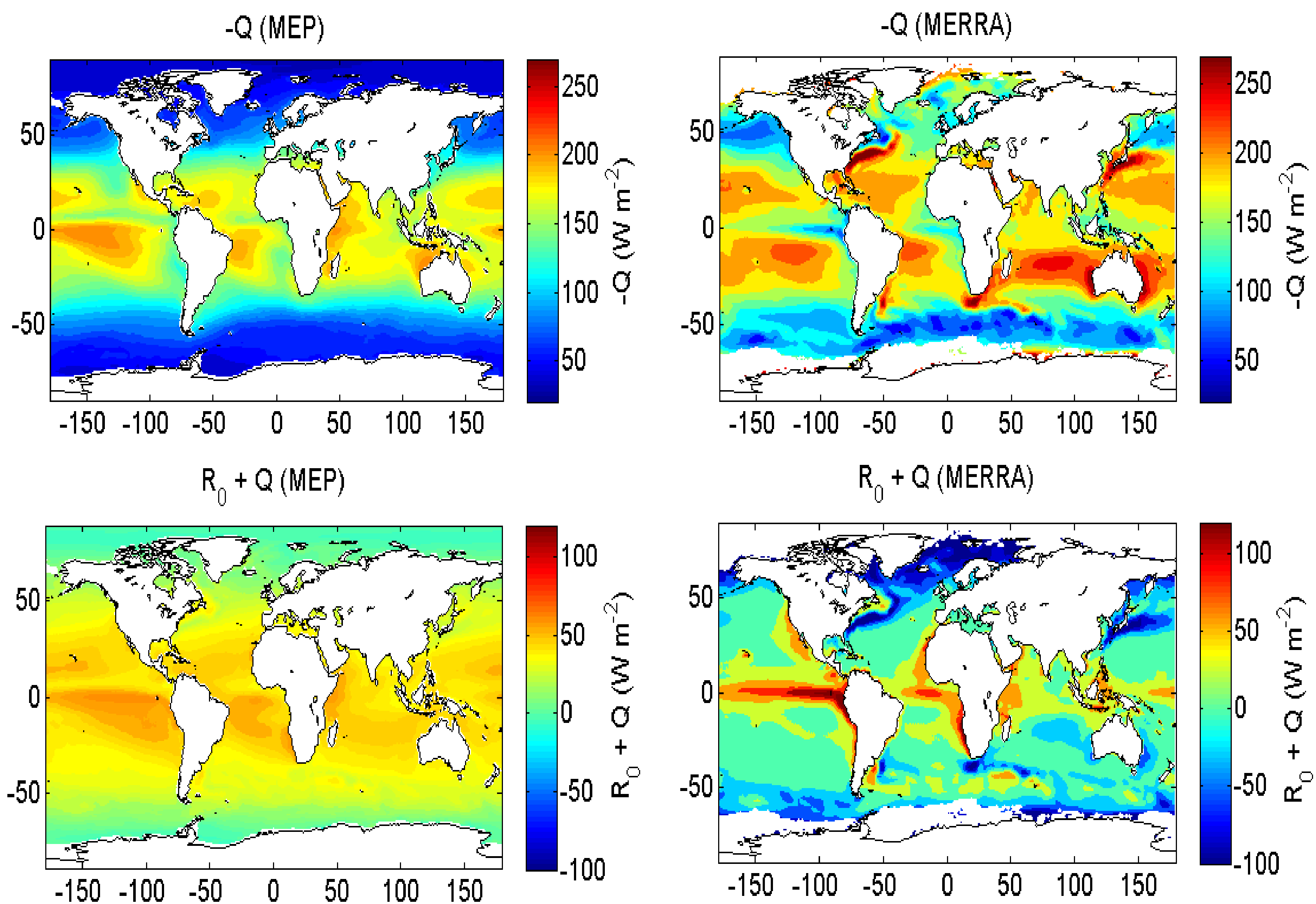
E (Observed) [Jung et al., 2010]



(*Huang et al, 2016*)



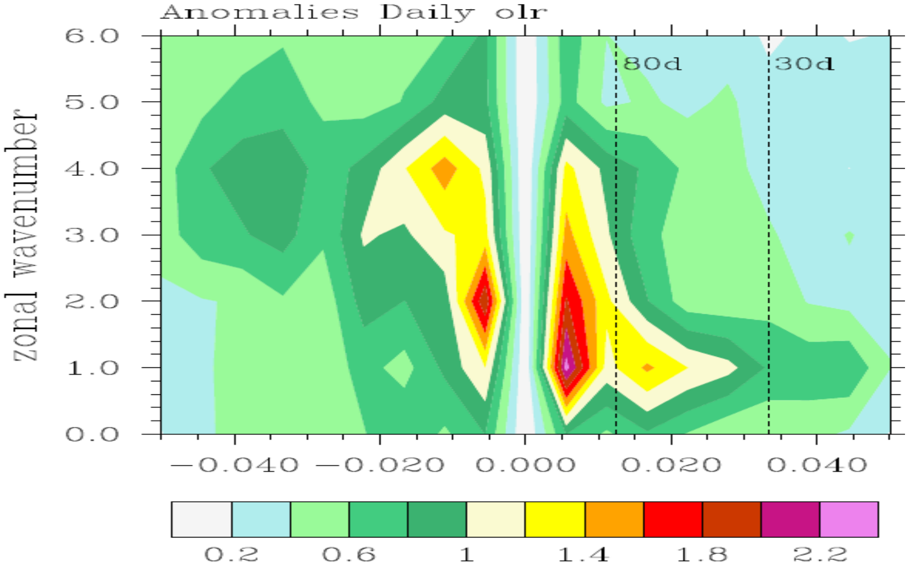
H (MEP)
H (MERRA)
H (W m^-2)
80
60
40
20
0
-20
50
0
-50
-150
-100
-50
0
50
100
150
Q (MEP)
Q (MERRA)
Q (W m^-2)
40
20
0
-20
-40
-60
1
0.5
0
-0.5
-1
-1.5
-2



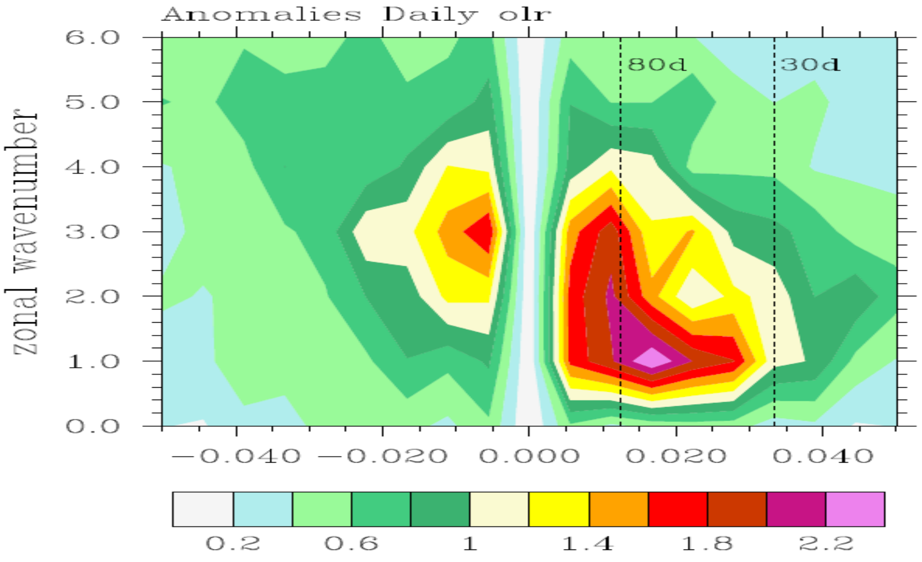
-Q (MEP)
-Q (MERRA)
R_0 + Q (MEP)
R_0 + Q (MERRA)
-Q (W m^{-2})
R_0 + Q (W m^{-2})
250
200
150
100
50
0
-50
-100
-150

Madden-Julian Oscillation (MJO)

bulk fluxes



MEP fluxes



Conclusions

- The information theory leads to a new approach of modeling the Earth surface energy budgets,
- The MEP model of surface energy budgets is an effective tool for monitoring and modeling global energy cycle,
- The MEP model provides a promising new physical scheme in the climate models,
- Other potential applications are yet to be explored.

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