Can causality be detected in noisy environmental systems?

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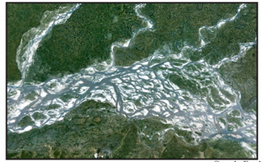
(1) Motivation

Are signals preserved in noisy systems?

Larsen et al. 2015 Photo: L. Hajek



Google Earth

- Detecting causality from data is a fundamental goal of computational science
- Complex systems can undergo critical transitions in response to external forcings or endogenic adjustments
- Detecting causality is necessary for anticipating critical transitions and developing robust predictions

(2) Background

Observational noise:
Error introduced through measurement (i.e. instrumental error).

Process noise:
Unexplained dynamics arising from high-dimensionality, complex interactions or stochasticity.

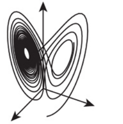
The Lorenz System

$$dX/dt = \sigma(Y - X)$$

$$dY/dt = X(\rho - Z)$$

$$dZ/dt = XY - \beta Z$$

$\sigma = 10$ (Prandtl #); $\rho = 28$ (Rayleigh #); $\beta = 8/3$ (Wave #)



Simulating Noise

Given a noise-free process, $S_t = F(S_{t-1})$, $S_t \in R^m$ observational noise results in:

$$X_t = h(S_t) + \epsilon_{obs}$$

where h is the observational function that maps points in R^m to R^l and ϵ_{obs} is the IID observational error. Similarly, process noise can be described as :

$$Z_t = G(Z_{t-1} + \epsilon_{process})$$

where $Z_t = G(Z_{t-1})$ is the noise-free map and $\epsilon_{process}$ is the error iterated through the system's dynamics.

Mutual information

$$I(x,y) = \sum_{x,y} p(x,y) \log_2 \frac{p(x,y)}{p(x)p(y)}$$

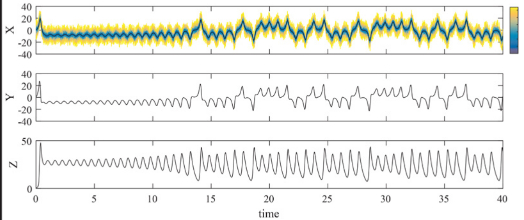
Transfer entropy

$$T_{x \to y} = \sum_{y_t y_t^k x_t^k} p(y_t, y_t^k, x_t^l) \log \left[\frac{p(y_t | y_t^k, x_t^l)}{p(y_t | y_t^k)} \right]$$

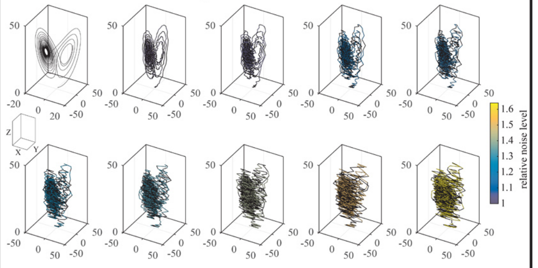
Ruddell, B. L., and P. Kumar. 2009. Ecohydrologic process networks: 1. Identification. Water Resources Research 45.
Larsen, L., L. Hajek, K. Maher, C. Paola, D. Merritts, T. Bralower, I. Montanez, S. Wing, N. Snyder, M. Hochella, L. R. Kump, and M. Person. 2015. Taking the pulse of Earth's surface systems. Eos.

(3) Observational Noise

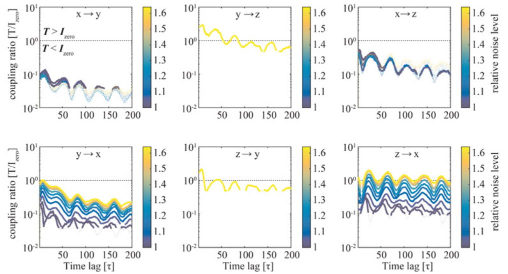
Observational noise added to X



Phase spaces across noise levels

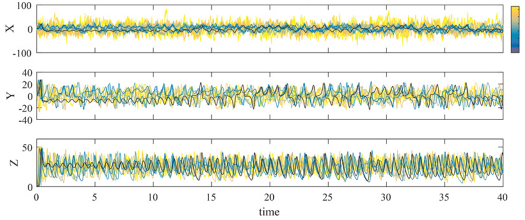


Information transfer across noise levels

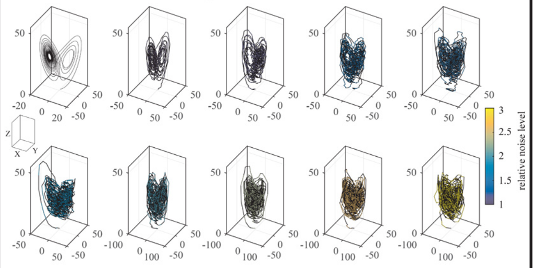


(4) Process Noise

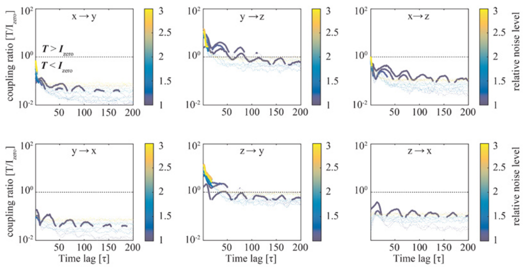
Process noise added to X, iterated to Y & Z



Phase spaces across noise levels

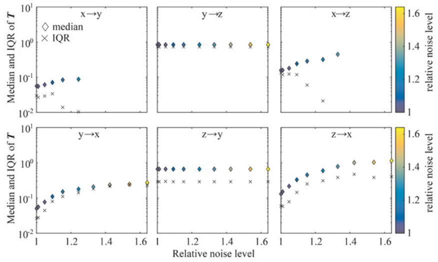


Information transfer across noise levels

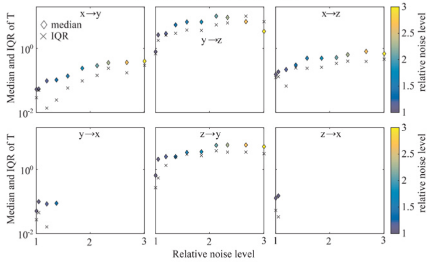


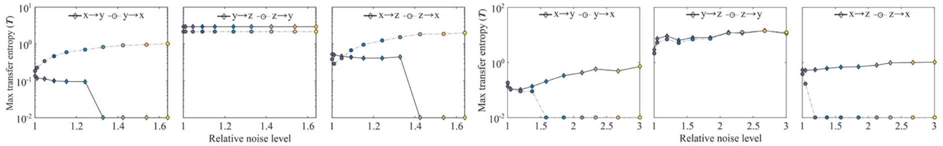
(5) Statistical Summary

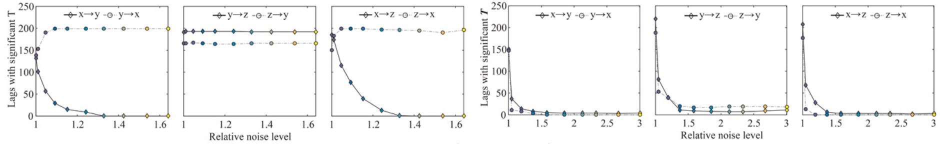
Observational Noise



Process Noise







Key Observations

Observational Noise

- With no noise Y & Z exhibit pairwise "forcing-like" behavior.
- As X's noise level increases it transfers less information to Y & Z.
- As X's noise level increases, Y & Z "remember" X's underlying dynamics and are able to reduce X's uncertainty.
- Changes in the central tendency & spread of pair-wise distributions of ***T*** exhibit functionally similar responses to increasing noise levels.

Process Noise

- With no noise Y & Z exhibit pairwise "forcing-like" behavior.
- As noise is added pair-wise ***T*** tends to increase but only occurs at short lags.
- At high noise levels ***T*** from Y and Z went to zero; X forces Y and Z.
- Again, changes in the central tendency & spread of individual pair-wise distributions of ***T*** exhibit functionally similar responses to increasing noise levels.