A Graph-Theoretic Approach to Delta Channel Networks: Entropic assessment of complexity

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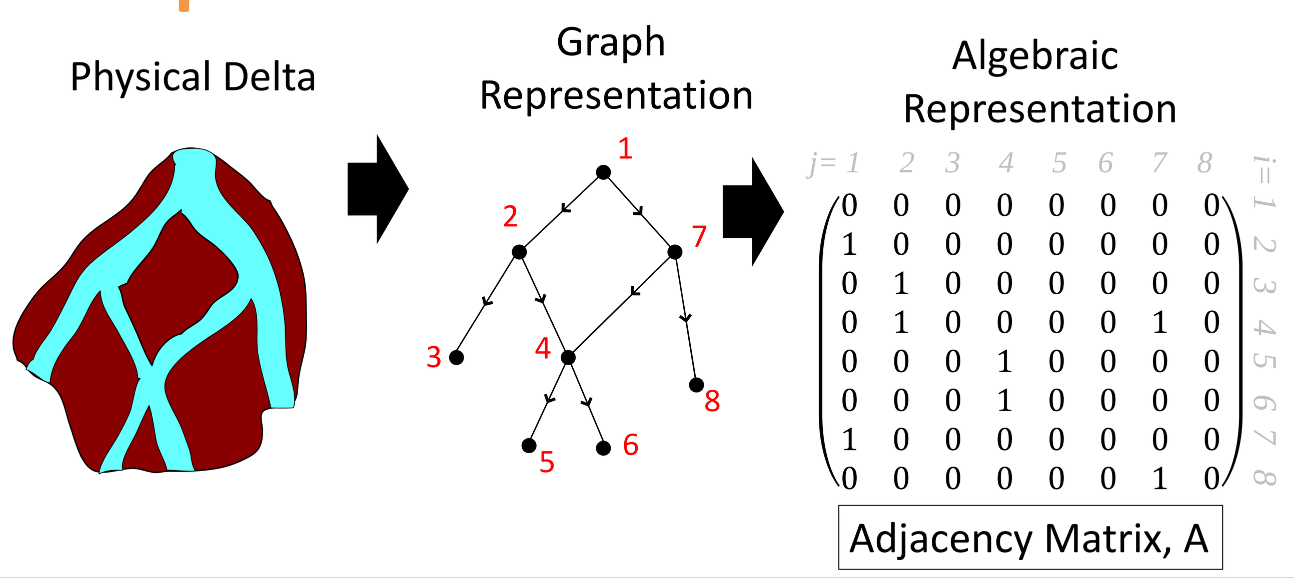
Goal

- **Quantify** the **patterns** imprinted on the landscape as a function of the **physical processes** that created them.
- Using the **graph theoretic framework** proposed by *Tejedor et al.* [2015a,b], we are able to construct **vulnerability** maps that depict the relative change of sediment and water delivery to the shoreline outlets in response to possible perturbations in hundreds of upstream links. We show that an **inverse relationship** exists between **entropy and vulnerability**, reinforcing the idea that entropy is a surrogate of the capacity of the system to undergo changes.
- Using **numerical models** (Delft3D), we quantify the **signature the incoming sediment size** on the structure and flux distribution of the emerging **delta channel network**.

Representation



Delta entropy

We conceptualize the **flow** in the delta by a large number of noninteracting flux packages that independently perform **random downstream walk** in a **directed graph** from the apex to the different outlets.

We set the **stationary probability** that the package of flux is traveling **from vertex *v* to vertex *u* proportional to the flux F_{uv}** at the link that connects these vertices:

$$p_{uv} \propto F_{uv}$$

The **joint entropy** of an apex-to-outlet subnetwork is given by:

$$H(u,v) = -\sum_{(vu)\in S_i} p_{uv} \log p_{uv}$$

$$= \underbrace{\sum_{(vu)\in S_i} p_{uv} \log \frac{p_{uv}}{p_{u\bullet} p_{\bullet v}}}_{I(u;v)} - \underbrace{\sum_{(vu)\in S_i} p_{uv} \log \frac{p_{uv}^2}{p_{u\bullet} p_{\bullet v}}}_{H(u|v)+H(v|u)}$$

Here we use Shannon entropy to measure the information content of channel splitting and rejoining in a delta.

The entropy can be interpreted as the ability of the system to undergo changes [*Ulanowicz et al.*, 2009] or in other words, it quantifies how the uncertainty of the system enables it to deal with perturbations.

Vulnerability Assessment

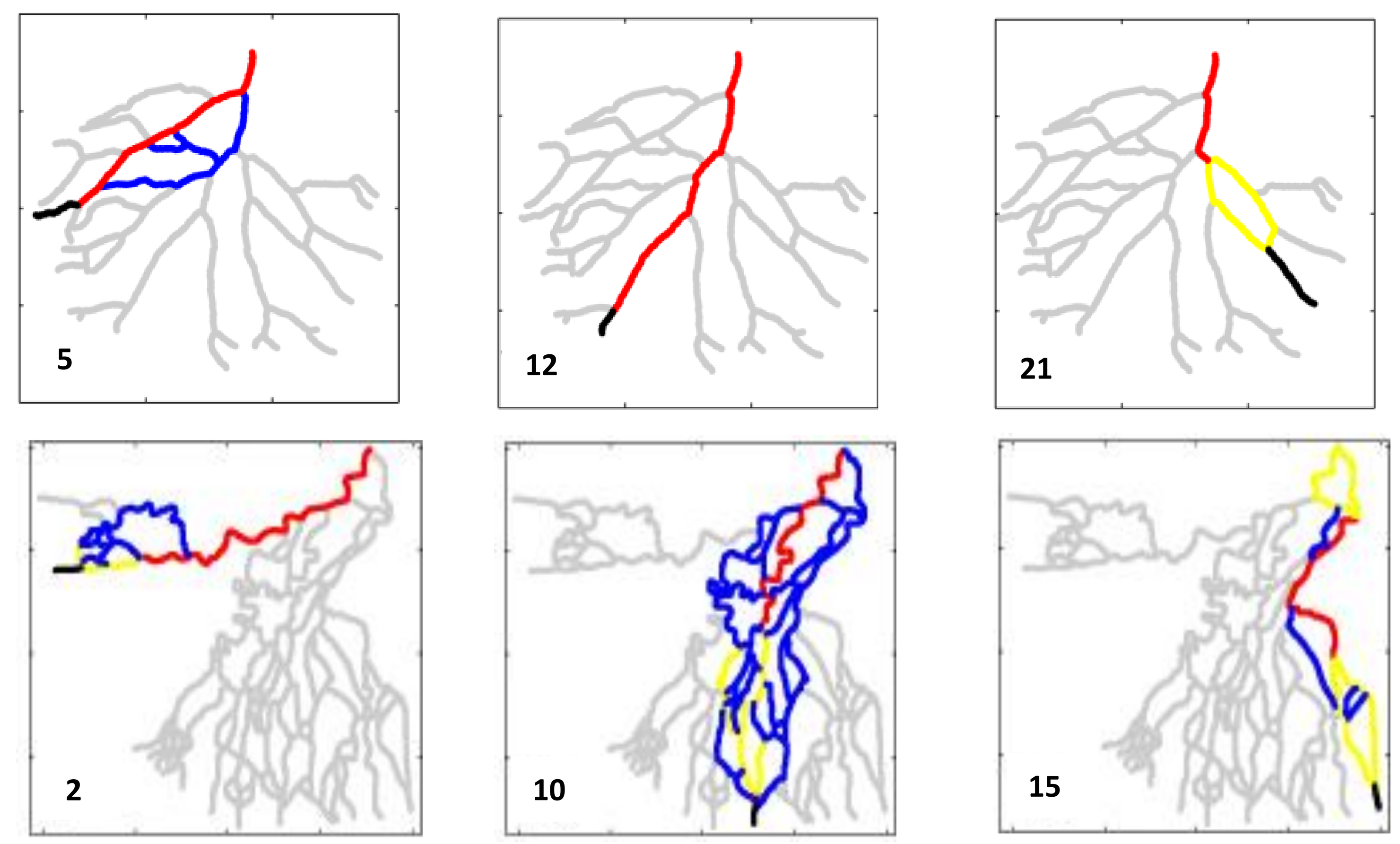
Vulnerability is defined in terms of how **changes** in **upstream** parts of the system would **affect** the **shoreline fluxes**. Questions such as what links of the delta network, if altered, might affect most drastically the distribution of fluxes to the costal outlets, or where an intervention should be imposed to maintain a desired flux to a particular outlet node for land building purposes, are important components of delta management towards sustainability.

We characterize the flux reduction at outlet *i* with respect to the flux reduction at link (*vu*) by the ***local vulnerability***:

$$V_{uv}^i = p_{uv}^i / C_{uv}^i$$

where p_{uv}^i is the fraction of the steady flux in link (*vu*) that drains to outlet *i* and C_{uv}^i is related to the ratio of the steady flux at the outlet F_i and the steady flux at the link (*vu*) F_{uv}.

Vulnerability maps



Each panel highlights the contributing network for a single outlet.. Red, yellow, and blue links represent high $(V_{uv}^i > 0.75)$, medium $(0.25 < V_{uv}^i \le 0.75)$, and low $(V_{uv}^i \le 0.25)$ values of the local vulnerability index. Shoreline outlets are shown in black.

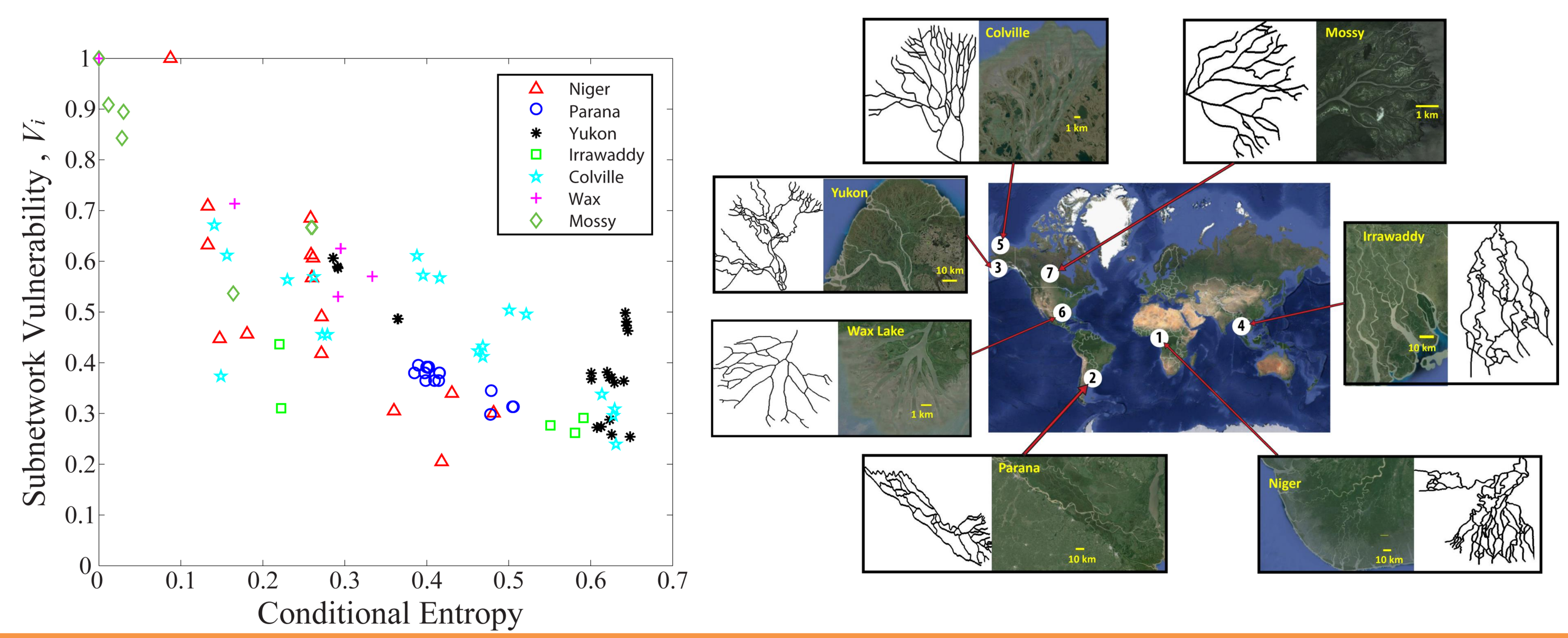
Entropy and Vulnerability

We can define the **subnetwork vulnerability** as the average of the local vulnerabilities over all channels (links) in the subnetwork that drains to outlet *i* :

$$V_i = \frac{1}{|E_i|} \sum_{(vu)\in E_i} V_{uv}^i$$

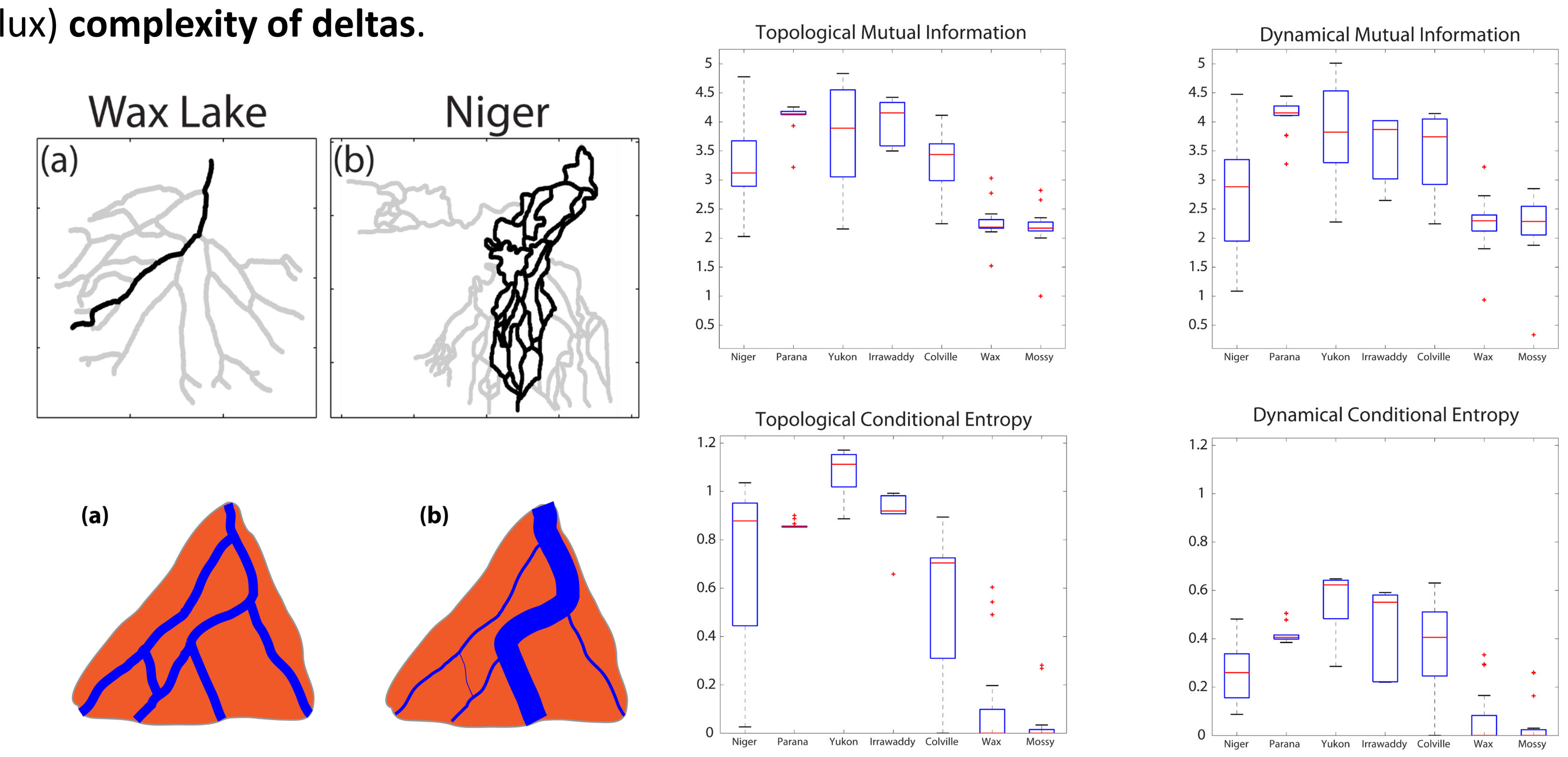
where $|E_i|$ denotes the number of links in the subnetwork that drains to outlet *i*.

We show that an **inverse relationship** exists between **entropy** and **subnetwork vulnerability**.



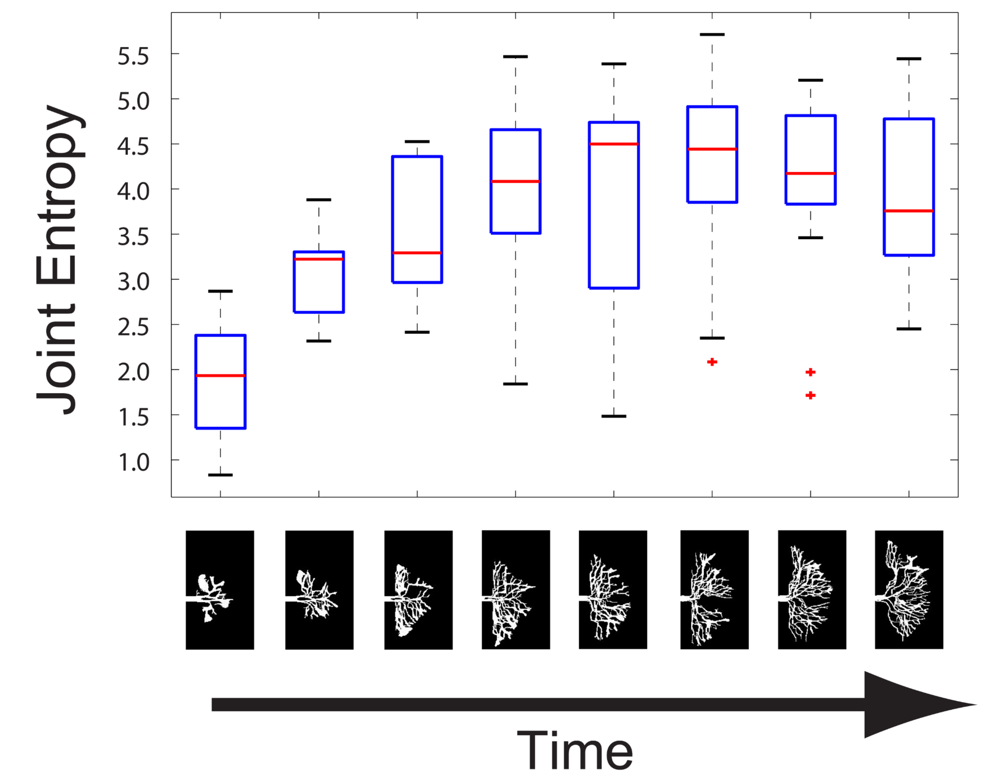
Quantifying Complexity

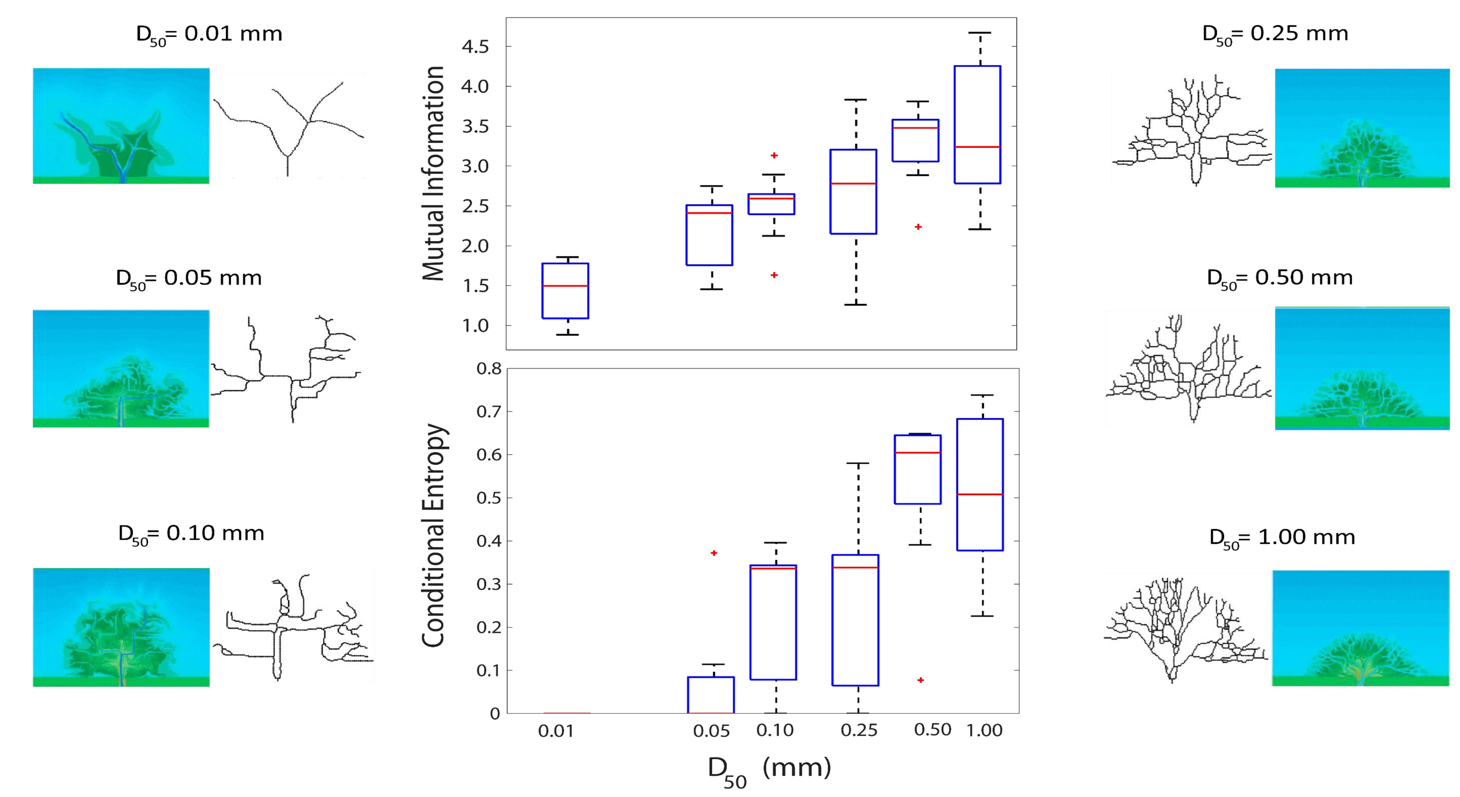
We define a **suite of metrics** that **depicts** the **topologic** (structure of pathways) and **dynamic** (flux) **complexity of deltas.**



Case study: Signature of sediment composition

- **Delft3D simulations** which reproduce a sediment-laden river entering a standing body of water that is **devoid of waves, tides, and buoyancy forces.** [*Caldwell and Edmonds*, 2014].
- Six runs of the model with **log-normally distributed incoming sediment** with a median size, D_{50} = 0.01 mm, 0.05 mm, 0.1 mm, 0.25 mm, 0.5 mm, 1 mm, and standard deviation $\sigma(\phi) = 1$ area analyzed here. (ϕ –space, where $\phi = -\log_2 D$)





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